September 12, 2025

National Chocolate Milkshake Day

Today in History:

Lascaux cave painting discovered (1940)

JFK marries Jacqueline Bouvier (1953)

Number of the Day: 9147

9147 = 3 x 3049

9147 is 6966 in base 11.

Fun Fact:

Houseflies hum in the key of F.

Quote of the Day:

"Sometimes I lie awake at night and ask why me? Then a voice answers nothing personal, your name just happened to come up."

Charles M. Schulz

Today's Weather:

Mainly sunny, high 75°

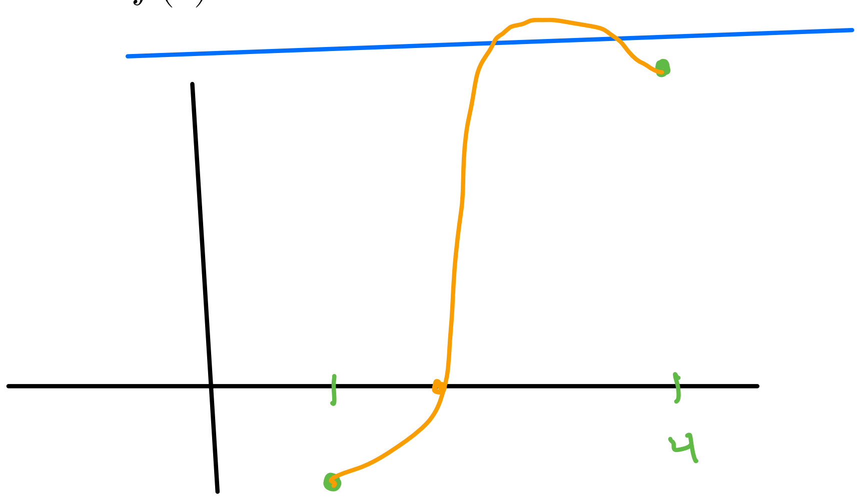
Math 121

Quiz #10

If $f(x)$ is continuous on $[1, 4]$ and $f(1) = -1$ and $f(4) = 6$.

For each of the following statements, say whether it is always true (AT), never true (NT), or sometimes true (ST):

1. $f(c) = 0$ has a solution with $1 < c < 4$ (AT)
2. $f(c) = 0$ has only one solution with $1 < c < 4$ (ST)
3. $f(c) = 7$ has a solution with $1 < c < 4$ (ST)



MPOD #10

PROB 6

$f(x) = x^3 - 3x^2 + 1 = 0$

x	$f(x)$
-3	-53
-2	-19
-1	-3
0	1
1	-1
2	-3
3	1

1

2

3

LIMIT

$$\lim_{x \to c} f(x) = L$$

FOR ALL $\varepsilon > 0$, THERE EXISTS $\delta > 0$
($\forall$) ($\exists$)

SUCH THAT IF
(S.T.)

$0 < |x - c| < \delta$ THEN

$|f(x) - L| < \varepsilon$

EXAMPLE PROVE

$$\lim_{x \to 2} (3x+1) = 7$$

$\varepsilon = .1$

$$|f(x) - L| < \varepsilon$$
$$|3x+1-7| < .1$$
$$|3x-6| < .1$$
$$|3(x-2)| < .1$$
$$|x-2| < \frac{.1}{3}$$
$$\downarrow$$
$$|x-c| < \delta$$

$$\delta = \frac{.1}{3}$$

$\varepsilon = .0001$

$$|f(x) - L| < \varepsilon$$
$$|3x+1-7| < .0001$$
$$|3x-6| < .0001$$
$$|3(x-2)| < .0001$$
$$|x-2| < \frac{.0001}{3} \qquad \delta = \frac{.0001}{3}$$

ε (VERY SMALL)

$$|f(x) - L| < \varepsilon \qquad |x-2| < \frac{\varepsilon}{3}$$
$$|3x+1-7| < \varepsilon$$
$$|3x-6| < \varepsilon \qquad \delta = \frac{\varepsilon}{3}$$
$$|3(x-2)| < \varepsilon$$

EXAMPLE 2 PROVE

$$\lim_{x \to 1} 2x+4 = 6$$

ε (VERY SMALL) $|f(x) - L| < \varepsilon$

$$|2x+4-6| < \varepsilon$$

$$|2x-2| < \varepsilon$$

$$|2(x-1)| < \varepsilon$$

$$|x-1| < \frac{\varepsilon}{2}$$

$$\delta = \frac{\varepsilon}{2}$$

WORK

PROOF

GIVEN $\varepsilon > 0$

LET $\delta = \frac{\varepsilon}{2}$

IF

$$0 < |x-1| < \delta$$

$$|x-1| < \frac{\varepsilon}{2}$$

$$|2(x-1)| < \varepsilon$$

$$|2x-2| < \varepsilon$$

$$|\underbrace{2x+4}_{f(x)} - \underbrace{6}_{L}| < \varepsilon$$

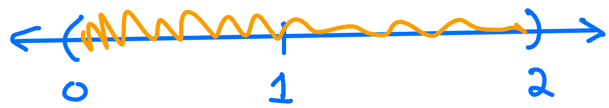
EXAMPLE 3

$$\lim_{x \to 1} x^2 = 1$$

ε (VERY SMALL)

$$|x^2 - 1| < \varepsilon$$

$$|(x-1)(x+1)| < \varepsilon$$



$x < 2$

$x + 1 < 3$

$$|(x-1)(x+1)| < |(x-1)(3)| < \varepsilon$$

$$|x-1| < \frac{\varepsilon}{3} \qquad \boxed{\delta = \frac{\varepsilon}{3}}$$

PROOF

GIVEN $\varepsilon > 0$ LET $\delta = \frac{\varepsilon}{3}$

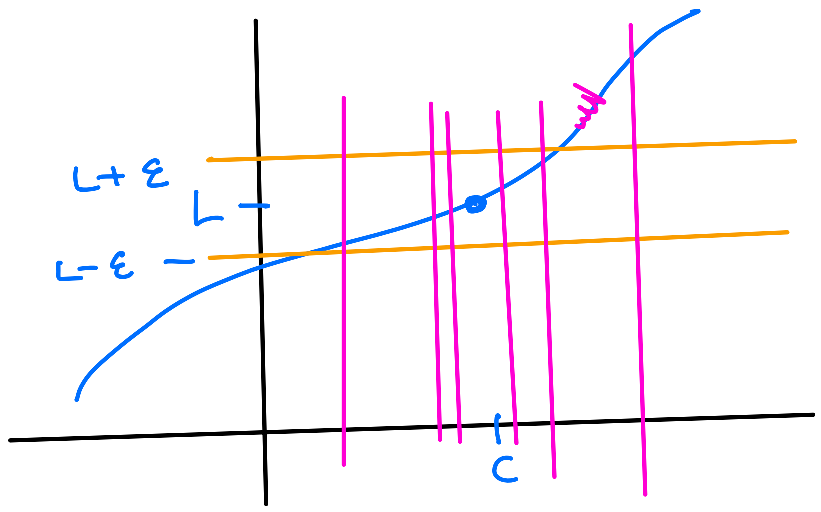
IF $0 < |x-1| < \delta$

$$0 < |x-1| < \frac{\varepsilon}{3}$$

$$0 < |3(x-1)| < \varepsilon$$

$$|(x-1)(x+1)| < \varepsilon \qquad \text{SINCE } \underline{(x+1) < 3}$$

$$|x^2 - 1| < \varepsilon$$



L+ε
L
L-ε
c