September 9, 2025

National Teddy Bear Day

Today in History:

Congress renames the nation “United States of America” (1776)

California becomes the 31st state (1850)

Number of the Day: 712

712 = 2 x 2 x 2 x 89

712 is the sum of the first 21 primes.

Fun Fact:

It is illegal for men to wear strapless dresses in Miami, Florida.

Quote of the Day:

"Don't stay in bed, unless you can make money in bed."

- George Burns

Today’s Weather:

Plenty of sunshine, high 76°.

Math 121

Quiz #8

Find

$$\lim_{x\to 0} \frac{x \sin x}{1-\cos x} \cdot \frac{(1+\cos x)}{(1+\cos x)}$$

$$= \lim_{x\to 0} \frac{x \sin x (1+\cos x)}{1-\cos^2 x} = \lim_{x\to 0} \frac{x \sin x (1+\cos x)}{\sin^2 x}$$

$$= \lim_{x\to 0} \left(\frac{x}{\sin x}\right) \cdot (1+\cos x) = 1 \cdot 2 = 2$$

$\left(\frac{x}{\sin x}\right) \to 1$, $(1+\cos x) \to 2$

Pg 99 #7

$$\lim_{t\to 2} (t^2-4)\cos\left(\frac{1}{t-2}\right) = 0$$

$$-1 \le \cos\left(\frac{1}{t-2}\right) \le 1$$

$$-(t^2-4) \le (t^2-4)\cos\left(\frac{1}{t-2}\right) \le (t^2-4)$$

$-(t^2-4) \to 0$, $(t^2-4) \to 0$

MPOD #5

$$\lim_{x\to 0}\frac{\sec x - 1}{x\sec x}$$

$$= \lim_{x\to 0}\frac{\frac{1}{\cos x} - 1}{\frac{x}{\cos x}} \cdot \frac{\cos x}{\cos x}$$

$$= \lim_{x\to 0}\frac{1-\cos x}{x}\cdot\frac{1+\cos x}{1+\cos x}$$

$$= \lim_{x\to 0}\frac{1-\cos^2 x}{x(1+\cos x)} = \lim_{x\to 0}\frac{\sin^2 x}{x(1+\cos x)}$$

$$= \lim_{x\to 0}\left(\frac{\sin x}{x}\right)\left(\frac{\sin x}{1}\right)\left(\frac{1}{(1+\cos x)}\right) = 1\cdot 0\cdot\frac{1}{2} = 0$$

$\frac{\sin x}{x} \to 1$, $\frac{\sin x}{1} \to 0$, $\frac{1}{(1+\cos x)} \to \frac{1}{2}$

(8) $$\lim_{x\to 0}\frac{1-\cos 3x}{2x^2}\left(\frac{1+\cos 3x}{1+\cos 3x}\right)$$

$$= \lim_{x\to 0}\frac{1-\cos^2 3x}{2x^2(1+\cos 3x)} = \lim_{x\to 0}\frac{\sin^2 3x}{2x^2(1+\cos 3x)}$$

$$= \lim_{x\to 0}\ 3\frac{\sin 3x}{3x}\ 3\frac{\sin 3x}{3x}\ \frac{1}{2}\ \frac{1}{1+\cos 3x}$$

$3\cdot 1\cdot 3\cdot 1\cdot\frac{1}{2}\cdot\frac{1}{2} = \frac{9}{4}$

HORIZONTAL ASYMPTOTES

$y = f(x)$ $\lim_{x \to +\infty} f(x) = L$ THEN $y = L$ IS A H.A.

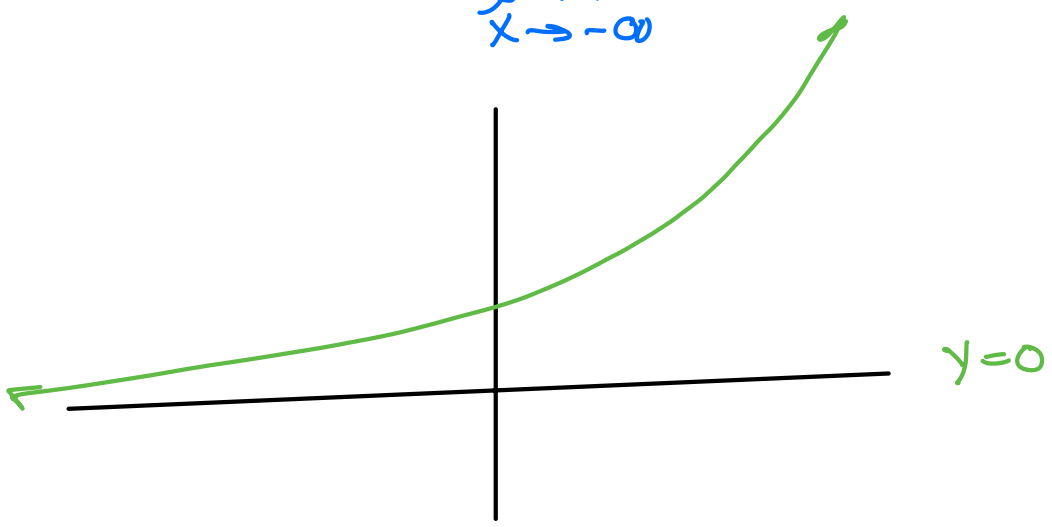
$\lim_{x \to -\infty} f(x) = M$ THEN $y = M$ IS A H.A.

EXAMPLE 1

$f(x) = e^x$

$\lim_{x \to +\infty} e^x = +\infty$

$\lim_{x \to -\infty} e^x = 0$

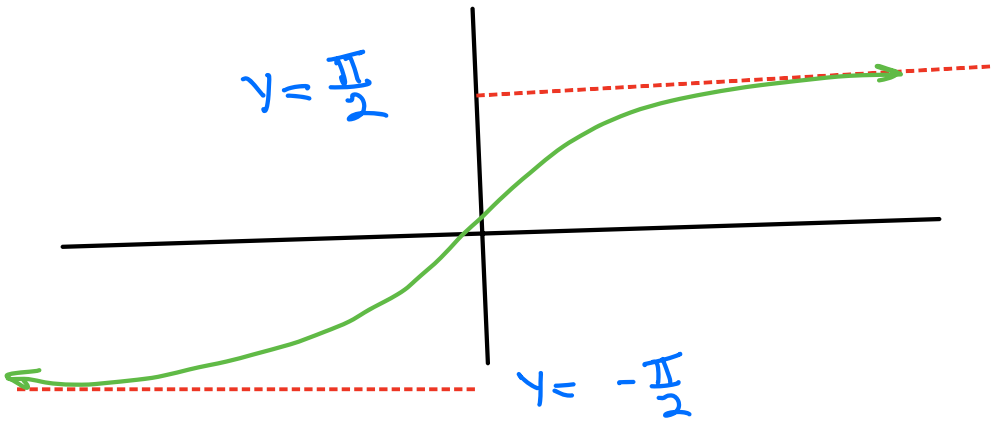


EXAMPLE 2

$f(x) = \text{ARCTAN}\, x$

$\lim_{x \to +\infty} f(x) = \frac{\pi}{2}$

$\lim_{x \to -\infty} f(x) = -\frac{\pi}{2}$

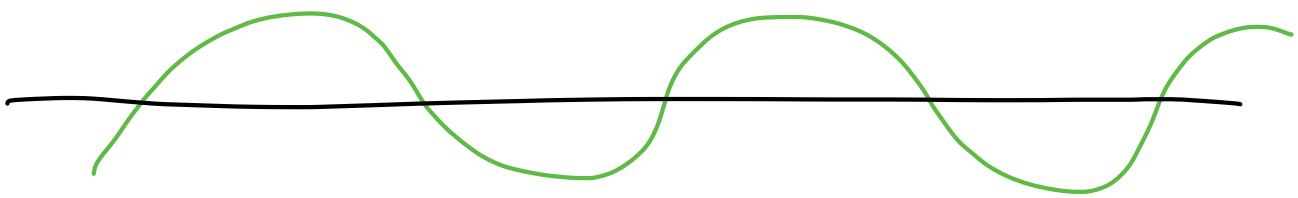


EXAMPLE 3

$f(x) = \sin x$ OR $\cos x$

$\lim_{x \to +\infty} f(x) = \text{DNE}$

$\lim_{x \to -\infty} f(x) = \text{DNE}$

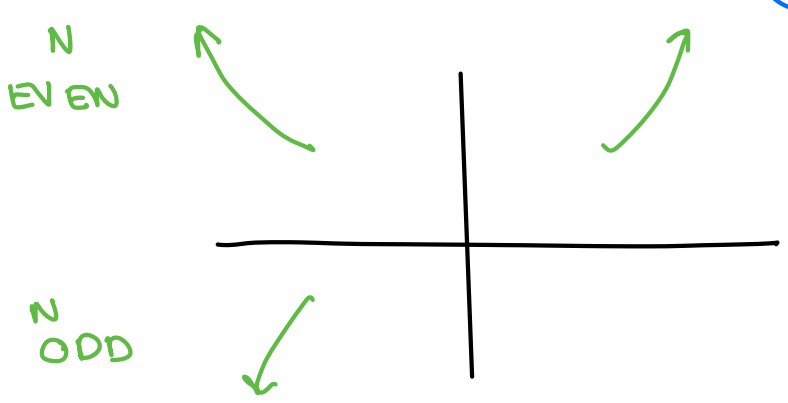


EXAMPLE 4

$f(x) = x^n$

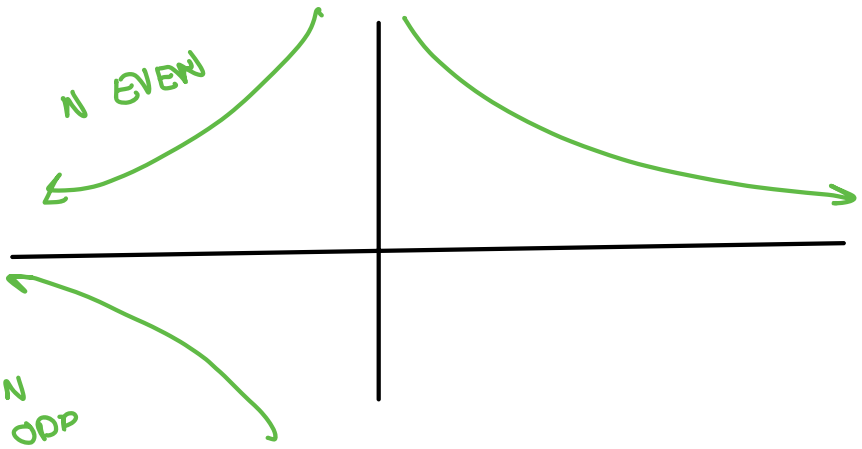
$\lim_{x \to +\infty} x^n = +\infty$

$\lim_{x \to -\infty} x^n = \begin{cases} +\infty & n \text{ EVEN} \\ -\infty & n \text{ ODD} \end{cases}$



EXAMPLE 5 $f(x) = \frac{1}{x^N}$

$\lim_{x \to +\infty} f(x) = 0$ $\lim_{x \to -\infty} f(x) = 0$



EXAMPLE 6 $f(x)$ IS A RATIONAL FUNCTION

$f(x) = \frac{P(x)}{Q(x)}$ $P(x), Q(x)$ ARE POLYNOMIALS

(1) $DEG(P(x)) < DEG(Q(x))$

$\lim_{x \to \pm\infty} f(x) = 0$ C.B.B.B.T.

(2) $DEG(P(x)) > DEG(Q(x))$

$\lim_{x \to \pm\infty} f(x) =$ DNE.

(3) $DEG(P(x)) = DEG(Q(x))$

$\lim_{x \to \pm\infty} f(x) = \frac{a}{b}$

a IS COOF. OF $P(x)$, b IS COOF OF $Q(x)$ (LARGEST POWER)

(a) $\lim_{x \to +\infty} \dfrac{2x+5}{3x^2+1} = 0$

(b) $\lim_{x \to \infty} \dfrac{2x^2+5}{3x^2+1} = \dfrac{2}{3}$

(c) $\lim_{x \to \infty} \dfrac{2x^3+5}{3x^2+1} = \text{DNE.}$

(d) $\lim_{x \to \infty} \dfrac{2x+1}{x - x\sqrt{x}} = 0$

(e) $\lim_{x \to -\infty} \sqrt{\dfrac{4x^2-5}{9x^2+x+1}}$

$= \sqrt{\lim_{x \to -\infty} \dfrac{4x^2-5}{9x^2+x+1}} = \sqrt{\dfrac{4}{9}} = \dfrac{2}{3}$

(f) $\lim_{x \to \infty} \dfrac{3x-2}{\sqrt{2x^2+1}} = \dfrac{3}{\sqrt{2}}$

$\lim_{x \to -\infty} \dfrac{3x-2}{\sqrt{2x^2+1}} = -\dfrac{3}{\sqrt{2}}$

(g) $\lim_{x \to \infty} \left(\sqrt{x^2+2x} - x\right)\left(\frac{\sqrt{x^2+2x} + x}{\sqrt{x^2+2x} + x}\right)$

$= \lim_{x \to \infty} \frac{x^2+2x-x^2}{\sqrt{x^2+2x}+x} = \frac{2}{2} = 1$

(h) $\lim_{x \to \infty} \frac{\sin x}{x} = 0$

$-\frac{1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$

$\frac{\sin x}{x}$ is squeezed; $-\frac{1}{x} \to 0$, $\frac{1}{x} \to 0$