

September 8, 2026

National Ampersand Day

Today in History:

Ford pardons Nixon (1974)

Oprah goes national (1986)

Number of the Day: 1590

1590 = $2 \times 3 \times 5 \times 53$

1590 is a number n such that $2n + 1$, $4n + 1$, and $8n + 1$ are all primes.

Fun Fact:

“Oreo’s” are the world’s best-selling brand of cookies (6 billion sold each year).

Quote of the Day:

“Life is like an ice-cream cone, you have to lick it one day at a time.”

- Charles M. Schulz

Today’s Weather:

Sun and clouds mixed, high near 83°

Math 121

Quiz #8

Find

$$\lim_{x \rightarrow 0} \frac{\sin x \sin 2x}{3x \sin 4x}$$

$$\lim_{x \rightarrow 0} \frac{\frac{\sin x}{x}}{1} \cdot \frac{\frac{\sin 2x}{2x}}{1} \cdot \frac{\frac{4x}{\sin 4x}}{1} \cdot \frac{\frac{2x}{3 \cdot 4x}}{\frac{2}{12}} = \frac{1}{6}$$

$$\# 2 \quad \lim_{x \rightarrow 0} \frac{1 - \cos 3x}{2x^2} \left(\frac{1 + \cos 3x}{1 + \cos 3x} \right)$$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos^2 3x}{2x^2 (1 + \cos 3x)} = \lim_{x \rightarrow 0} \frac{\sin^2 3x}{2x^2 (1 + \cos 3x)}$$

$$= \lim_{x \rightarrow 0} \frac{\frac{\sin 3x}{3x}}{1} \cdot \frac{\frac{\sin 3x}{3x}}{1} \cdot \frac{1}{2} \cdot \frac{1 \cdot 3 \cdot 3}{(1 + \cos 3x)} = \frac{1}{2} \cdot \frac{9}{2} = \frac{9}{4}$$

$$\approx 7 \quad \lim_{x \rightarrow 0} \frac{\sec 2x \tan 2x}{x}$$

$$= \lim_{x \rightarrow 0} \frac{1}{\cos 2x} \cdot \frac{\sin 2x}{\cos 2x} \cdot \frac{2}{2x}$$

$\downarrow$
 $1 \cdot 1 \cdot 2 = 2$

HORIZONTAL ASYMPTOTES

LIMITS AT $\pm$ INFINITY

$$y = f(x)$$

$$\left[\begin{array}{l} \lim_{x \rightarrow +\infty} f(x) = L \Rightarrow y = L \text{ is a H.A.} \\ \lim_{x \rightarrow -\infty} f(x) = K \Rightarrow y = K \text{ is a H.A.} \end{array} \right.$$

EXAMPLE: $f(x) = e^x$

$$\lim_{x \rightarrow \infty} e^x = +\infty$$

NO ASYMPTOTE

$$\lim_{x \rightarrow -\infty} e^x = 0$$

$y = 0$ is a H.A.

EXAMPLE 2 $f(x) = \sin x / \cos x$

$$\lim_{x \rightarrow \infty} \sin x = \text{DNE}$$

$$\lim_{x \rightarrow -\infty} \sin x = \text{DNE}$$

EXAMPLE 3

$$f(x) = x^N$$

$$\lim_{x \rightarrow \infty} x^N = +\infty \quad \lim_{x \rightarrow -\infty} x^N = \begin{cases} +\infty & N \text{ EVEN} \\ -\infty & N \text{ ODD} \end{cases}$$

EXAMPLE 4

$$f(x) = \frac{1}{x^N}$$

$$\lim_{x \rightarrow \pm\infty} \frac{1}{x^N} = 0 \quad \text{C.B.B.T.}$$

$y=0$ is a H.A.

EXAMPLE 5

$$f(x) = \frac{P(x)}{Q(x)}$$

RATIONAL FUNCTION

$$\textcircled{1} \text{ DEG}(P(x)) < \text{DEG}(Q(x))$$

$$\lim_{x \rightarrow \pm\infty} \frac{P(x)}{Q(x)} = 0$$

$$\textcircled{2} \text{ DEG}(P(x)) > \text{DEG}(Q(x))$$

$$\lim_{x \rightarrow \pm\infty} \frac{P(x)}{Q(x)} = \begin{matrix} +\infty \\ -\infty \end{matrix} \left. \vphantom{\lim_{x \rightarrow \pm\infty} \frac{P(x)}{Q(x)}} \right\} \text{DNE.}$$

$$\textcircled{3} \text{ DEG}(P(x)) = \text{DEG}(Q(x))$$

$$\lim_{x \rightarrow \pm\infty} \frac{P(x)}{Q(x)} = \frac{a}{b}$$

a IS LEADING COEFF OF $P(x)$

b IS LEADING COEFF OF $Q(x)$

$$\lim_{x \rightarrow \infty} \frac{2x+5}{3x^2+1} = 0$$

$$\lim_{x \rightarrow -\infty} \frac{2x^3+5}{3x^2+1} = \text{DNE}$$

$$\lim_{x \rightarrow \infty} \frac{2x^2+5}{3x^2+1} = \frac{2}{3}$$

$$\lim_{x \rightarrow \infty} \frac{2x+1}{x-x\sqrt{x}} = 0$$

$$\begin{aligned} \lim_{x \rightarrow -\infty} \sqrt{\frac{4x^2-5}{9x^2+x+1}} &= \sqrt{\lim_{x \rightarrow -\infty} \frac{4x^2-5}{9x^2+x+1}} \\ &= \sqrt{\frac{4}{9}} = \frac{2}{3} \end{aligned}$$

$$\lim_{x \rightarrow \infty} \frac{3x-2}{\sqrt{2x^2+1}} = \frac{3}{\sqrt{2}}$$

$$\lim_{x \rightarrow -\infty} \frac{3x-2}{\sqrt{2x^2+1}} = -\frac{3}{\sqrt{2}}$$

$$\lim_{x \rightarrow \infty} \sqrt{x^2 + 2x} - x \left(\frac{\sqrt{x^2 + 2x} + x}{\sqrt{x^2 + 2x} + x} \right)$$

$$= \lim_{x \rightarrow \infty} \frac{\cancel{x^2} + 2x - \cancel{x^2}}{\sqrt{x^2 + 2x} + \cancel{x}} = \frac{2}{2} = 1$$

$$\lim_{x \rightarrow \infty} \frac{\sin x}{x} = 0$$

$$\frac{-1}{x} \leq \frac{\sin x}{x} \leq \frac{1}{x}$$

$$\left(\text{REMEMBER} \right. \lim_{x \rightarrow 0} \frac{\sin x}{x} = 1 \left. \right)$$

$$\lim_{x \rightarrow \infty} \frac{x - \cos x}{x}$$

$$\lim_{x \rightarrow \infty} \frac{x}{x} - \frac{\cos x}{x} = \lim_{x \rightarrow \infty} 1 - \frac{\cos x}{x}$$

$$1 - 0 = 1$$