

September 8, 2025

National Ampersand Day

Today in History:

Ford pardons Nixon (1974)

Oprah goes national (1986)

Number of the Day: 1590

1590 = $2 \times 3 \times 5 \times 53$

1590 is a number n such that $2n + 1$, $4n + 1$, and $8n + 1$ are all primes.

Fun Fact:

“Oreo’s” are the world’s best-selling brand of cookies (6 billion sold each year).

Quote of the Day:

“Life is like an ice-cream cone, you have to lick it one day at a time.”

- Charles M. Schulz

Today’s Weather:

Sun and clouds mixed, high near 68°

Math 121

Quiz #7

Find

$$\lim_{x \rightarrow 8} \frac{x - 8}{\sqrt{x - 4} - 2} \left(\frac{\sqrt{x - 4} + 2}{\sqrt{x - 4} + 2} \right)$$

$$= \lim_{x \rightarrow 8} \frac{(x - 8)(\sqrt{x - 4} + 2)}{x - 4 - 4}$$

$$= \lim_{x \rightarrow 8} \frac{\cancel{(x - 8)}(\sqrt{x - 4} + 2)}{\cancel{(x - 8)}} = 4$$

$$\textcircled{31} \quad \lim_{t \rightarrow 2} \frac{2^{2t} + 2^t - 20}{2^t - 4} \quad x = 2^t$$

$$\lim_{t \rightarrow 2} \frac{x^2 + x - 20}{x - 4} = \lim_{t \rightarrow 2} \frac{\cancel{(x - 4)}(x + 5)}{\cancel{(x - 4)}}$$

$$= \lim_{t \rightarrow 2} 2^t + 5 = 9$$

$$\textcircled{2)} \lim_{h \rightarrow 0} \frac{\sqrt{2+h} - 2}{h} = \text{D.N.E.}$$

$$\text{m.p.o.d.} \quad \lim_{x \rightarrow 1} \left(\frac{1}{x-1} - \frac{2}{x^2-1} \right)$$

$$= \lim_{x \rightarrow 1} \left(\frac{1}{x-1} \cdot \frac{(x+1)}{(x+1)} - \frac{2}{(x^2-1)} \right)$$

$$= \lim_{x \rightarrow 1} \left(\frac{x+1}{x^2-1} - \frac{2}{(x^2-1)} \right) = \lim_{x \rightarrow 1} \frac{\cancel{x-1}}{\cancel{(x-1)}(x+1)}$$

$$= \frac{1}{2}$$

TRIG LIMITS

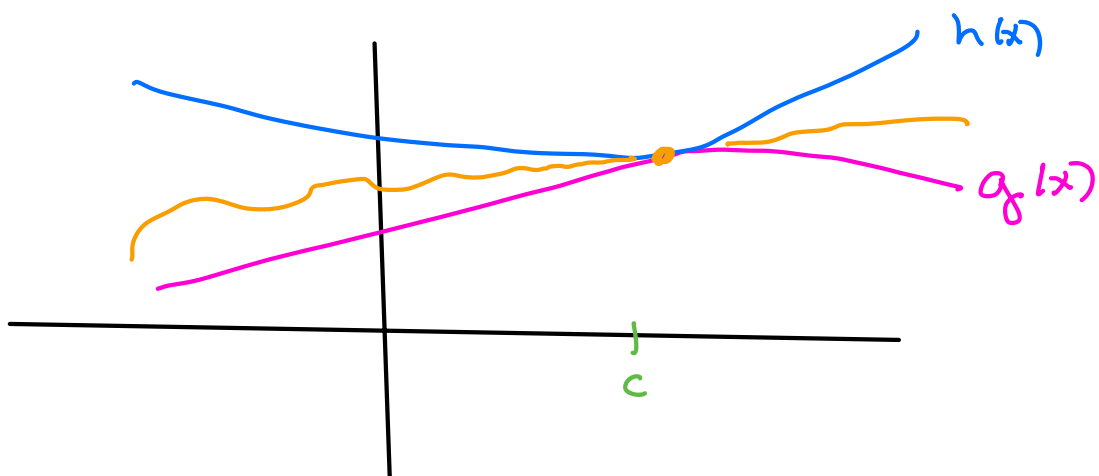
SQUEEZE THM / SANDWICH THM / POLICE THM

$$\text{IF } g(x) \leq f(x) \leq h(x)$$

$$\lim_{x \rightarrow c} g(x) = L$$

$$\lim_{x \rightarrow c} h(x) = L$$

$$\text{THEN } \lim_{x \rightarrow c} f(x) = L$$



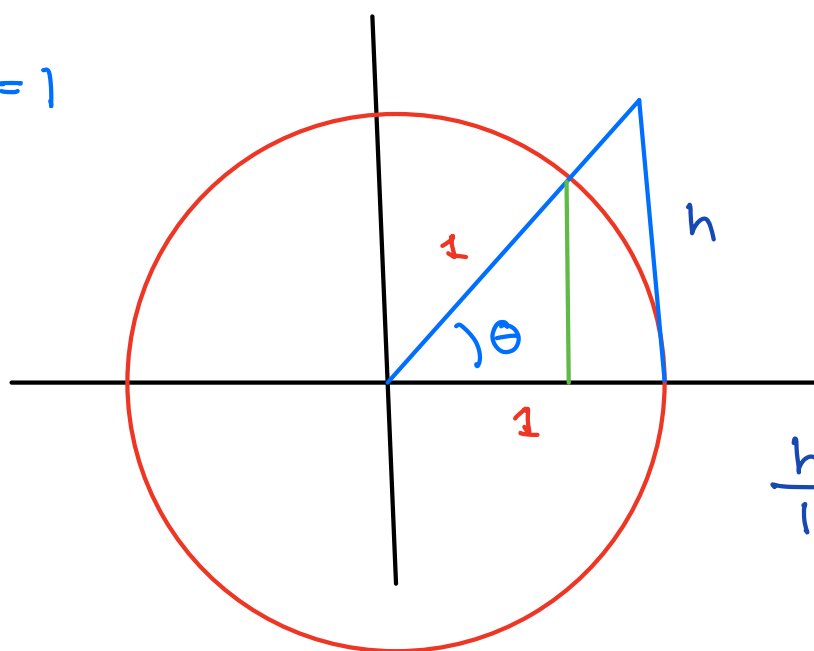
$$\lim_{x \rightarrow 0} x \sin\left(\frac{1}{x}\right) = 0$$

$$x \left(-1 \leq \sin\left(\frac{1}{x}\right) \leq 1 \right)$$

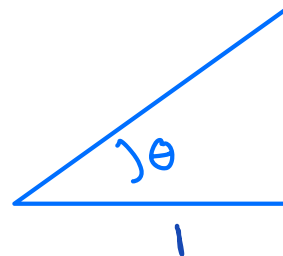
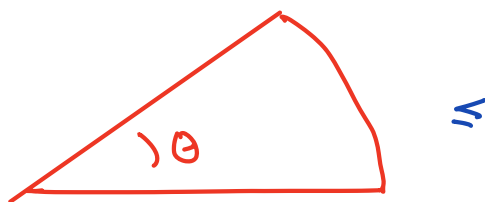
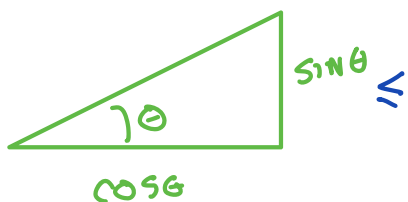
$$-x \leq x \sin\left(\frac{1}{x}\right) \leq x$$

Two green arrows point from the left and right sides of the inequality to the origin (0) on the x-axis.

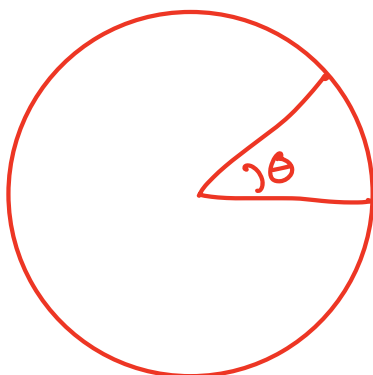
$$x^2 + y^2 = 1$$



$$\frac{h}{1} = \tan \theta$$



$$\frac{1}{2} \cos \theta \sin \theta \leq \frac{\theta}{2} \leq \frac{1}{2} \tan \theta$$



$$\frac{A}{\theta} = \frac{\text{AREA CIRCLE}}{2\pi}$$

$$\frac{A}{\theta} = \frac{\pi(1)^2}{2\pi}$$

$$A = \frac{1}{2} \theta$$

$$\frac{1}{2} \cos \theta \sin \theta \leq \frac{\theta}{2} \leq \frac{\tan \theta}{2}$$

$$\frac{\cos \theta \sin \theta}{\sin \theta} \leq \frac{\theta}{\sin \theta} \leq \frac{\sin \theta}{\cos \theta \sin \theta}$$

$$\cos \theta \leq \frac{\theta}{\sin \theta} \leq \frac{1}{\cos \theta}$$

$$\frac{1}{\cos \theta} \geq \frac{\sin \theta}{\theta} \geq \cos \theta$$

$$\cos \theta \leq \frac{\sin \theta}{\theta} \leq \frac{1}{\cos \theta}$$

\downarrow \downarrow
 1 1

$$\lim_{\theta \rightarrow 0} \frac{\sin \theta}{\theta} = 1$$

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{y \rightarrow 0} \frac{\sin y}{y} = 1$$

$$\lim_{t \rightarrow 0} \frac{\sin t}{t} = 1$$

$$\lim_{s \rightarrow 0} \frac{\sin(s)}{s} = 1$$

EXAMPLE

$$\textcircled{1} \lim_{x \rightarrow 0} \left[\frac{\sin(3x)}{3x} \right] = 1$$

$$\textcircled{2} \lim_{x \rightarrow 0} \frac{1 - \cos x}{x} \left(\frac{1 + \cos x}{1 + \cos x} \right)$$

$$= \lim_{x \rightarrow 0} \frac{1 - \cos^2 x}{x(1 + \cos x)} = \lim_{x \rightarrow 0} \frac{\sin^2 x}{x(1 + \cos x)}$$

$$= \lim_{x \rightarrow 0} \left(\frac{\sin x}{x} \right) \left(\frac{\sin x}{1} \right) \left(\frac{1}{1 + \cos x} \right) = 0$$

\downarrow 1 \downarrow 0 \downarrow $\frac{1}{2}$

$$\textcircled{3} \lim_{x \rightarrow 0} \frac{\sin(2x) \sin(3x)}{x^2}$$

$$= \lim_{x \rightarrow 0} 2 \left(\frac{\sin(2x)}{2x} \right) \cdot 3 \left(\frac{\sin(3x)}{3x} \right) = 2 \cdot 3 = 6$$

$$\textcircled{4} \lim_{x \rightarrow 0} \frac{x}{\tan(5x)} = \lim_{x \rightarrow 0} \frac{x}{\frac{\sin(5x)}{\cos(5x)}}$$

$$= \lim_{x \rightarrow 0} 5 \left(\frac{5x}{\sin(5x)} \right) \frac{\cos(5x)}{1} = \frac{1}{5}$$

TWO LIMITS YOU NEED
TO KNOW:

$$\lim_{x \rightarrow 0} \frac{\sin x}{x} = 1$$

$$\lim_{x \rightarrow 0} \frac{1 - \cos x}{x} = 0$$