

# **October 6, 2025**

## **National Noodle Day**

### **Today in History:**

First U.S. Train robbery (1866)

Jane Eyre is published (1847)

### **Number of the Day: 1833**

$$\mathbf{1833} = 3 \times 13 \times 47$$

**1833** is the start of the first run of exactly 7 consecutive odd composite numbers.

### **Fun Fact:**

The Beatles performed their first U.S. concert in Carnegie Hall.

### **Quote of the Day:**

“Just Play. Have Fun. Enjoy the Game.”

Michael Jordan

### **Today's Weather:**

Sun and clouds mixed, high 81°

# Math 121

## Quiz #21

Find  $f'(x)$  for

$$f(x) = x^2 \arctan(e^x)$$

$$f'(x) = x^2 \left( \frac{e^x}{1 + (e^x)^2} \right) + 2x \operatorname{ARCTAN}(e^x)$$

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mpoo 2)  
\*3  $y = \operatorname{ARCSIN}\left(\frac{x}{3}\right)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1 - \left(\frac{x}{3}\right)^2}} \cdot \frac{1}{3}$$

$$= \frac{1}{\sqrt{1 - \frac{x^2}{9}}} \cdot \frac{1}{3}$$

$$= \frac{1}{\sqrt{1 - \frac{x^2}{9}}} \cdot \frac{1}{\sqrt{9}}$$

$$= \frac{1}{\sqrt{(1 - \frac{x^2}{9})9}} = \frac{1}{\sqrt{9 - x^2}}$$

$$*4 \quad y = \sec(2x) - \operatorname{arcsec}\left(\frac{x}{2}\right)$$

$$\frac{dy}{dx} = 2\sec(2x)\tan(2x) - \frac{1}{|\frac{x}{2}| \sqrt{\left(\frac{x}{2}\right)^2 - 1}} \cdot \frac{1}{2}$$

$$= 2\sec(2x)\tan(2x) - \frac{1}{|x| \sqrt{\frac{x^2}{4} - \frac{4}{4}}}$$

$$= 2\sec(2x)\tan(2x) - \frac{1}{|x| \sqrt{\frac{x^2 - 4}{4}}}$$

$$= 2\sec(2x)\tan(2x) - \frac{2}{|x| \sqrt{x^2 - 4}}$$


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WE KNOW

$$y = e^x \quad \frac{dy}{dx} = e^x$$

$$e^y = e^{\ln x}$$

$$\underline{e^y} = e^{\ln x} = \underline{x}$$

$$\frac{d}{dx} [e^y = x]$$

$$e^y \frac{dy}{dx} = 1 \quad \frac{dy}{dx} = \frac{1}{e^y} = \frac{1}{x}$$

$$f(x) = \ln x \quad f'(x) = \frac{1}{x}$$

$$f(x) = \ln(\text{STUFF}) \quad f'(x) = \frac{\text{STUFF}'}{\text{STUFF}}$$

### EXAMPLE

$$f(x) = \ln(x^2 + 3x + 5)$$

$$f'(x) = \frac{2x + 3}{x^2 + 3x + 5}$$

$$f(x) = x^2 \ln x$$

$$f'(x) = x^2 \left( \frac{1}{x} \right) + 2x \ln x$$

$$= x + 2x \ln x$$

$$y = \ln(x^2) \quad y' = \frac{2x}{x^2} = \frac{2}{x}$$

↙  
 $y = 2 \ln x \quad y' = 2 \left( \frac{1}{x} \right)$

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$$\underline{\underline{y = b^x}} \quad \underline{\underline{\ln y = \ln b^x = x \ln b}}$$

$$\frac{1}{y} \frac{dy}{dx} = \ln b \quad \frac{dy}{dx} = y \ln b$$

$$y = b^x \quad \frac{dy}{dx} = b^x \ln b$$

$$y = b^{\text{JUNK}} \quad \frac{dy}{dx} = b^{\text{JUNK}} \ln b \cdot \text{JUNK}'$$


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EXAMPLE

$$y = 3^x \quad \frac{dy}{dx} = 3^x \ln 3$$

$$y = 4^{5x} \quad \frac{dy}{dx} = 4^{5x} \ln 4 \cdot 5$$

$$y = 5^{\sin x} \quad \frac{dy}{dx} = 5^{\sin x} (\ln 5) (\cos x)$$


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$$\ln x = \log_e x$$

$$y = \log_b x \iff b^y = x$$

$$b^y \ln b \frac{dy}{dx} = 1$$

$$\frac{dy}{dx} = \frac{1}{b^y \ln b} = \frac{1}{x \ln b}$$

$$y = \log_b x$$

$$\frac{dy}{dx} = \frac{1}{x \ln b}$$

$$y = \log_b (\text{JUNK})$$

$$\frac{dy}{dx} = \frac{1}{\text{JUNK} \ln b} \cdot \text{JUNK}'$$

$$y = \log_3 x \quad \frac{dy}{dx} = \frac{1}{x \ln 3}$$

Example

$$y = \ln \left( \frac{(3x^4 + 4x^3 + 7)^5 (2x + 1)^6}{\sqrt{5x + \sin x}} \right)$$

$$y = 5 \ln(3x^4 + 4x^3 + 7) + 6 \ln(2x + 1) - \frac{1}{2} \ln(5x + \sin x)$$

$$\frac{dy}{dx} = 5 \frac{12x^3 + 12x^2}{3x^4 + 4x^3 + 7} + 6 \frac{2}{2x + 1} - \frac{1}{2} \frac{5 + \cos x}{5x + \sin x}$$

$$\ln y = \ln \left( \frac{(2x^2 + x + 5)^5 (x^7 - x^6)^3}{(\cos x + e^x)^2} \right)$$

$$\ln y = 5 \ln(2x^2 + x + 5) + 3 \ln(x^7 - x^6) - 2 \ln(\cos x + e^x)$$

$$\frac{1}{y} \frac{dy}{dx} = 5 \frac{4x + 1}{2x^2 + x + 5} + 3 \frac{7x^6 - 6x^5}{x^7 - x^6} - 2 \frac{-\sin x + e^x}{\cos x + e^x}$$

$$\frac{dy}{dx} = y \left( 5 \frac{4x + 1}{2x^2 + x + 5} + 3 \frac{7x^6 - 6x^5}{x^7 - x^6} - 2 \frac{-\sin x + e^x}{\cos x + e^x} \right)$$

$$= \left( \frac{(2x^2 + x + 5)^5 (x^7 - x^6)^3}{(\cos x + e^x)^2} \right) \left( 5 \frac{4x + 1}{2x^2 + x + 5} + 3 \frac{7x^6 - 6x^5}{x^7 - x^6} - 2 \frac{-\sin x + e^x}{\cos x + e^x} \right)$$

$$y = \underline{x^x}$$

LOG. DIFF.

WHAT H KNOW:

$$y = x^n$$

$$\frac{dy}{dx} = nx^{n-1}$$

$$y = b^x$$

$$\frac{dy}{dx} = b^x \ln b$$

$$\underline{y = x^x}$$

$$[\ln y = \ln x^x = \underline{x \ln x}]$$

$$\frac{1}{y} \frac{dy}{dx} = x \left( \frac{1}{x} \right) + \ln x$$

$$\frac{dy}{dx} = y (1 + \ln x)$$

$$\frac{dy}{dx} = x^x (1 + \ln x)$$

$$\underline{y = x^{\sin x}}$$

$$\ln y = \ln x^{\sin x} = \underline{\sin x \ln x}$$

$$\frac{1}{y} \frac{dy}{dx} = \sin x \left( \frac{1}{x} \right) + (\ln x)(\cos x)$$

$$\frac{dy}{dx} = y \left[ \frac{\sin x}{x} + (\ln x)(\cos x) \right]$$

$$\frac{dy}{dx} = x^{\sin x} \left[ \frac{\sin x}{x} + (\ln x)(\cos x) \right]$$

$$\frac{f(x)}{e^x}$$

$$\ln x$$

$$e^{\text{JUNK}}$$

$$\ln(\text{JUNK})$$

$$b^x$$

$$b^{\text{JUNK}}$$

$$\log_b x$$

$$\log_b(\text{JUNK})$$

$$(\text{FUNCTION})^{\text{FUNCTION}}$$

$$\frac{f'(x)}{e^x}$$

$$\frac{1}{x}$$

$$e^{\text{JUNK}} \cdot \text{JUNK}'$$

$$\frac{\text{JUNK}'}{\text{JUNK}}$$

$$b^x \ln b$$

$$b^{\text{JUNK}} (\ln b)(\text{JUNK})'$$

$$\frac{1}{x \ln b}$$

$$\frac{1}{\text{JUNK} \ln b} \cdot \text{JUNK}'$$

MUST USE LOG DIFF.