November 4, 2025

Use your Common Sense Day

Today in History:

Entrance to King Tut's tomb discovered (1922)

Abraham Lincoln marries Mary Todd (1842)

Number of the Day: 2044

2044 = 2 x 2 x 7 x 73

2044 is the sum of the cubes of the divisors of 12

Fun Fact:

Morphine was names after Morpheus, the Greek god of dreams.

Quote of the Day:

Courage is what it takes to stand up and speak; courage is also what it takes to sit down and listen.

-Winston Churchill

Today's Weather:

Partly cloudy, high near 58^o.

Math 121 - Quiz #34

Find

$$\sum_{j=1}^{30}(2j+1)$$

$$= \sum_{j=1}^{30} 2j + \sum_{j=1}^{30} 1 = 2\sum_{j=1}^{30} j + \sum_{j=1}^{30} 1$$

$$= \cancel{2}\left(\frac{30(31)}{\cancel{2}}\right) + 30 = 930 + 30 = 960$$

$$3+5+7+9+11+13+ \cdots + 61$$

MPOD 34

#8

$$\lim_{N\to\infty} \sum_{j=1}^{N} \left(1+\frac{2j}{N}\right)^2 \underline{\left(\frac{2}{N}\right)}$$

$$= \lim_{N\to\infty} \frac{2}{N} \sum_{j=1}^{N} \left(1 + \frac{4j}{N} + \frac{4j^2}{N^2}\right)$$

$$= \lim_{N\to\infty} \frac{2}{N}\left[\sum_{j=1}^{N} 1 + \frac{4}{N}\sum_{j=1}^{N} j + \frac{4}{N^2}\sum_{N=1}^{N} j^2\right]$$

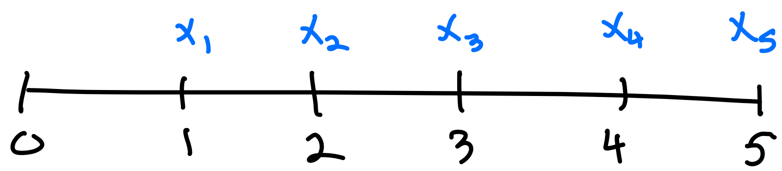
$$= \lim_{N\to\infty} \frac{2}{N}\left[N + \frac{4}{N}\,\frac{N(N+1)}{2} + \frac{4}{N^2}\,\frac{N(N+1)(2N+1)}{6}\right]$$

$$= \lim_{N\to\infty} \left[2 + \frac{8}{N^2}\,\frac{N(N+1)}{2} + \frac{8}{N^3}\,\frac{N(N+1)(2N+1)}{6}\right]$$

$$= 2 + \frac{8}{2} + \frac{16}{6} = \frac{26}{3}$$

4 $\quad f(x) = \sqrt{x} \qquad [0,5] \qquad N = 5$

$$\Delta x = \frac{b-a}{N} = \frac{5-0}{5} = 1$$



$$\sum_{j=1}^{5} f(x_j)\,\Delta x = f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x + f(x_5)\Delta x$$

$$= (\sqrt{1})(1) + (\sqrt{2})(1) + (\sqrt{3})(1) + (\sqrt{4})(1) + (\sqrt{5})(1)$$

$y = f(x) \qquad N\text{-SUB.} \qquad [a,b]$

RIGHT RIEMANN SUM

$$R_N = \sum_{j=1}^{N} f(x_j)\,\Delta x \qquad \Delta x = \frac{b-a}{N}$$

$$x_j = a + j\,\Delta x$$

$$\lim_{N\to\infty} R_N = \lim_{N\to\infty} \sum_{j=1}^{N} f(x_j)\,\Delta x = \text{AREA}$$

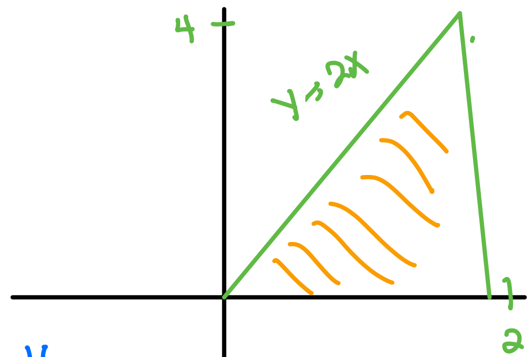
$$\lim_{N\to\infty}\sum_{j=1}^{N} f(x_j)\,\Delta x = \int_a^b f(x)\,dx$$

DEFINITE INTEGRAL

$$\int_a^b f(x)\,dx = \text{NET SIGNED AREA}$$

EXAMPLE 1

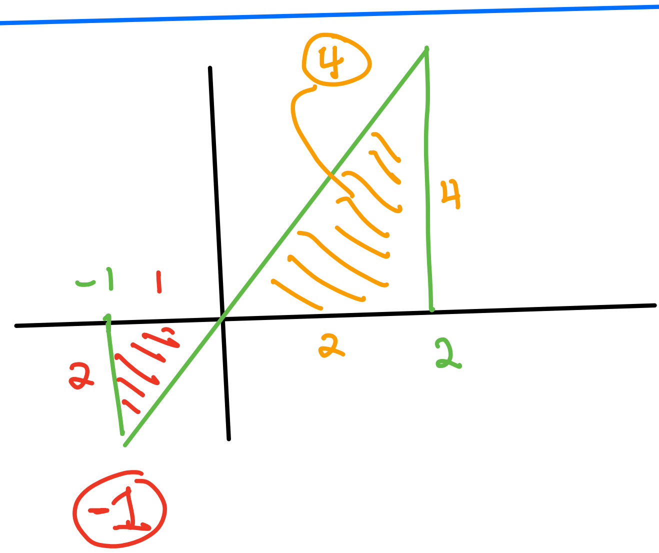
$$\int_0^2 2x\,dx$$



$$= \frac{1}{2}(2)(4) = 4$$

EXAMPLE 2

$$\int_{-1}^{2} 2x\,dx$$

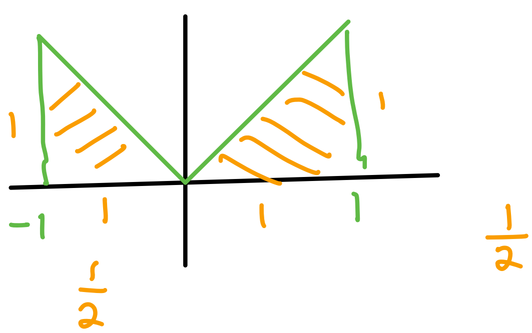


$$= 4 - 1 = 3$$

EXAMPLE 3

$$\int_{-1}^{1} |x|\,dx = \frac{1}{2} + \frac{1}{2}$$

$$= 1$$



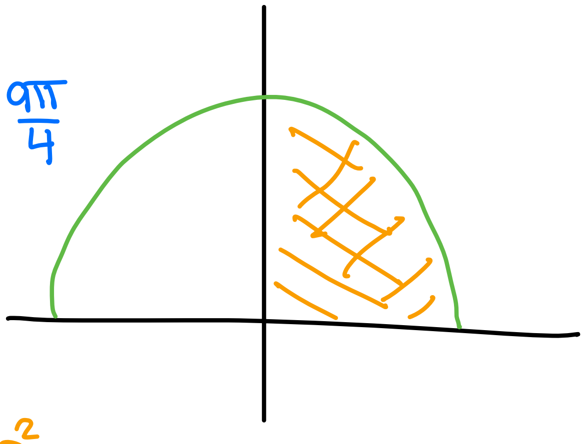
EXAMPLE 4

$$\int_0^3 \sqrt{9 - x^2}\, dx = \frac{9\pi}{4}$$

$y = \sqrt{9 - x^2}$

$y^2 = 9 - x^2$

$x^2 + y^2 = 9$

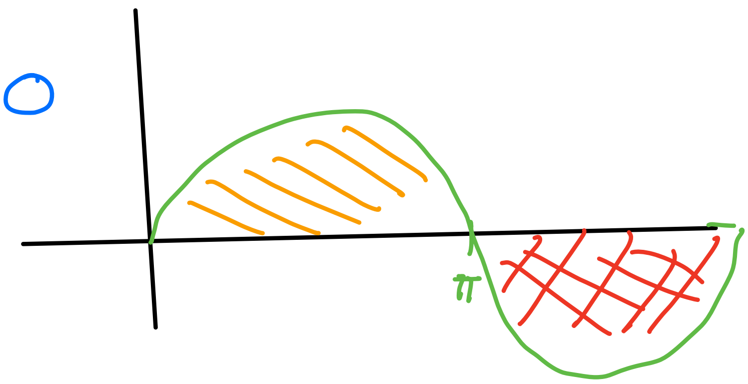


πR^2

$\pi (3)^2 = 9\pi$

EXAMPLE 5

$$\int_0^{2\pi} \sin x \, dx = 0$$



RULES

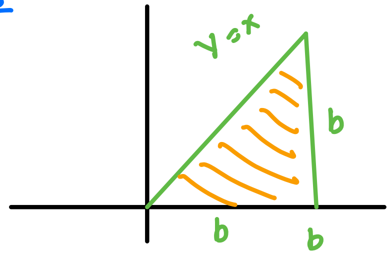
① $\displaystyle\int_a^b (f(x) \pm g(x))\, dx = \int_a^b f(x)\, dx \pm \int_a^b g(x)\, dx$

② $\displaystyle\int_a^b c f(x)\, dx = c \int_a^b f(x)\, dx$

③ $\displaystyle\int_a^b f(x)\, dx = - \int_b^a f(x)\, dx$

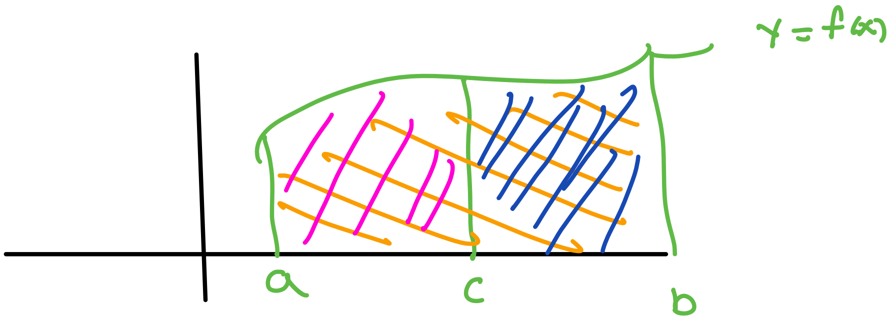
④ $\displaystyle\int_a^a f(x)\, dx = 0$

5 $\int_0^b x\,dx = \frac{1}{2}b^2$



6 $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)\,dx$

$a < c < b$

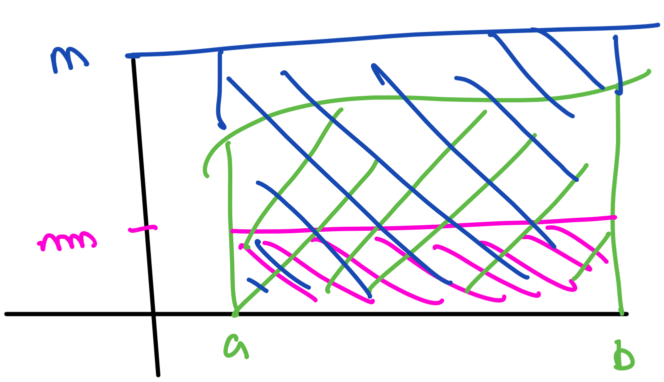


7 IF $f(x) < g(x)$

$\int_a^b f(x)\,dx < \int_a^b g(x)\,dx$

8 IF $m < f(x) < M$ THEN

$m(b-a) < \int_a^b f(x)\,dx < M(b-a)$



EXAMPLE 17 $\quad f(x) = x^2 \quad [0,b] \quad$ N-SUB.

$$\int_0^b x^2\,dx = \lim_{N\to\infty} \sum_{j=1}^{N} f(x_j)\,\Delta x$$

$$\Delta x = \frac{b-a}{N} = \frac{b-0}{N} = \frac{b}{N}$$

$$x_j = a + j\,\Delta x = 0 + j\left(\frac{b}{N}\right) = \frac{bj}{N}$$

$$= \lim_{N\to\infty} \sum_{j=1}^{N} f\left(\frac{bj}{N}\right)\left(\frac{b}{N}\right)$$

$$= \lim_{N\to\infty} \sum_{j=1}^{N} \frac{b^2 j^2}{N^2}\,\frac{b}{N} = \lim_{N\to\infty} \frac{b^3}{N^3} \sum_{j=1}^{N} j^2$$

$$= \lim_{N\to\infty} \frac{b^3}{N^3}\,\frac{N(N+1)(2N+1)}{6} = \frac{2b^3}{6} = \frac{b^3}{3}$$

EXAMPLE (-6) $\quad f(x) = x^3 \quad [0,b] \quad$ N-SUB

$$\int_0^b x^3\,dx = \lim_{N\to\infty} \sum_{j=1}^{N} f(x_j)\,\Delta x$$

$$\Delta x = \frac{b-a}{N} = \frac{b-0}{N} = \frac{b}{N}$$

$$= \lim_{N\to\infty} \sum_{j=1}^{N} f\left(\frac{jb}{N}\right)\frac{b}{N}$$

$$x_j = a + j\,\Delta x = j\left(\frac{b}{N}\right)$$

$$= \lim_{N\to\infty} \sum_{j=1}^{N} \left(\frac{jb}{N}\right)^3 \frac{b}{N}$$

$$= \lim_{N\to\infty} \frac{b^4}{N^4} \sum_{j=1}^{N} j^3 = \lim_{N\to\infty} \frac{b^4}{N^4}\left(\frac{N(N+1)}{2}\right)^2$$

$$= \frac{b^4}{4}$$

$$\int_0^b x^1\,dx = \frac{b^2}{2}$$

$$\int_0^b x^2\,dx = \frac{b^3}{3}$$

$$\int_0^b x^3\,dx = \frac{b^4}{4}$$