November 3, 2025
National Sandwich Day

Today in History:

D.C. residents cast first presidential votes (1964)

Panama declares independence (1903)

Number of the Day: 1130

1130 = 2 x 5 x 113

1130 is the product of three distinct Sophie Germain primes.

Fun Fact:

600 cows are used to manufacture a seasons worth of NFL footballs.

Quote of the Day:

"It's just a job. Grass grows, birds fly, waves pound the sand. I beat people up."

-Muhammad Ali

Today's Weather:

Windy with a few showers early, high of 54^{o}.

Math 121 - Quiz #33

For the function

$$f(x) = x^3 - x - 1$$

1. Write the Newton's Method formula.

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \qquad x_{n+1} = x_n - \frac{x_n^3 - x_n - 1}{3x_n^2 - 1}$$

2. Find x_1 if $x_0 = 1$

$$f(1) = 1 - 1 - 1 = -1$$

$$f'(1) = 3 - 1 = 2$$

$$x_1 = 1 - \frac{-1}{2} = 1.5$$

SIGMA NOTATION

$$a_1 + a_2 + a_3 + a_4 + \dots + a_n = \sum_{j=1}^{n} a_j$$

EXAMPLE

$$\sum_{j=1}^{5} 2j = 2 + 4 + 6 + 8 + 10 = 30$$

$$\sum_{j=1}^{3} \sqrt{j} = \sqrt{1} + \sqrt{2} + \sqrt{3}$$

$$\sum_{j=1}^{4} j^3 = 1 + 8 + 27 + 64$$

RULES

$$\sum_{j=1}^{N} (a_j + b_j) = \sum_{j=1}^{N} a_j + \sum_{j=1}^{N} b_j$$

$$\sum_{j=1}^{N} c a_j = c \sum_{j=1}^{N} a_j$$

$$\sum_{j=1}^{N} c = \overbrace{c + c + c + \ldots + c}^{N} = cN$$

$$\sum_{j=1}^{N} j = 1 + 2 + 3 + 4 + \ldots + N = \frac{N(N+1)}{2}$$

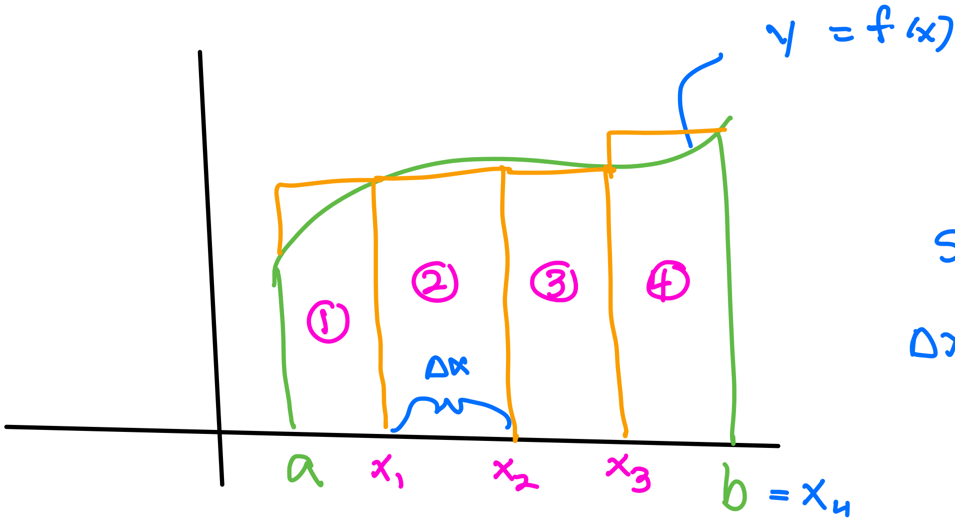
EXAMPLE $\sum_{j=1}^{10} j = \underbrace{1 + 2 + 3 + 4 + \ldots + 10} = \frac{10(11)}{2} = 55$

$$\sum_{j=1}^{N} j^2 = 1 + 4 + 9 + 16 + \ldots + N^2 = \frac{N(N+1)(2N+1)}{6}$$

EXAMPLE $\sum_{j=1}^{4} j^2 = 1 + 4 + 9 + 16 = \frac{4(5)(9)}{6} = 30$

$$\sum_{j=1}^{N} j^3 = 1 + 8 + 27 + \ldots + N^3 = \left(\frac{N(N+1)}{2}\right)^2$$

AREA



SUB-INTERVALS

$$\Delta x = \frac{b-a}{N}$$

$A_1 = f(x_1)\Delta x$ $\quad$ $A_2 = f(x_2)\Delta x$

$A_3 = f(x_3)\Delta x$ $\quad$ $A_4 = f(b)\Delta x$

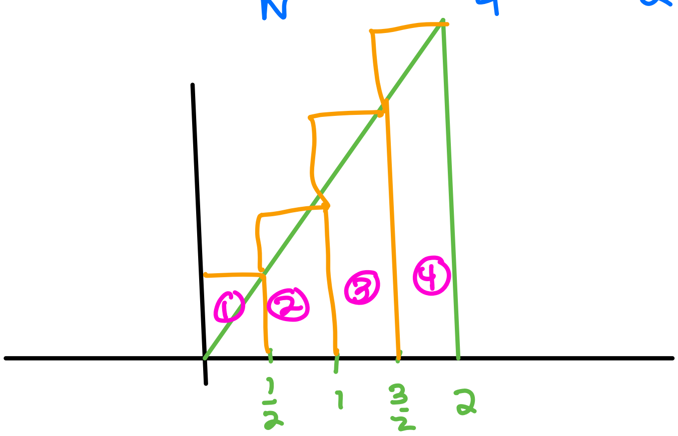
$$A \approx f(x_1)\Delta x + f(x_2)\Delta x + f(x_3)\Delta x + f(x_4)\Delta x$$

$$\underline{\left(f(x_1) + f(x_2) + f(x_3) + f(x_4)\right)}\Delta x$$

$$\sum_{j=1}^{4} f(x_j)\Delta x$$

EXAMPLE $\quad f(x) = 2x \quad [0,2] \quad N = 4$ SUB-INT.

$$\Delta x = \frac{b-a}{N} = \frac{2-0}{4} = \frac{1}{2}$$



$$A_1 = f(\tfrac{1}{2})\Delta x = 1(\tfrac{1}{2}) = \tfrac{1}{2}$$

$$A_2 = f(1)\Delta x = 2(\tfrac{1}{2}) = 1$$

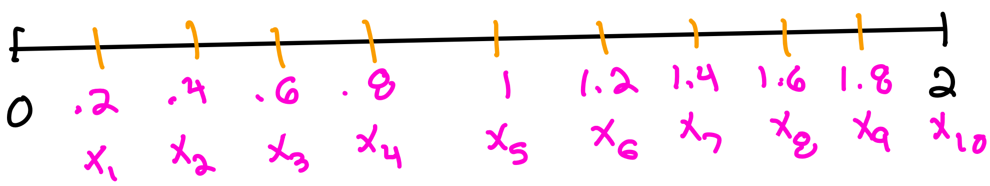
$$A_3 = f(\tfrac{3}{2})\Delta x = 3(\tfrac{1}{2}) = \tfrac{3}{2}$$

$$A_4 = f(2)\Delta x = 4(\tfrac{1}{2}) = 2$$

$$A \approx 5$$

$f(x) = 2x$ $\quad [0,2] \quad N=10$

$$\Delta x = \frac{b-a}{N} = \frac{2-0}{10} = .2$$



$$x_j = a + j\Delta x = 0 + j(.2) = .2j$$

$$A \approx \sum_{j=1}^{10} f(x_j)\Delta x = \sum_{j=1}^{10} f(.2j)\Delta x$$

$$\sum_{j=1}^{10} (2)(.2j)(.2) = .08 \sum_{j=1}^{10} j$$

$$= (.08)\left(\frac{10(11)}{2}\right) = 4.4$$

$f(x) = 2x \qquad [0,2] \qquad N\text{-SUB.}$

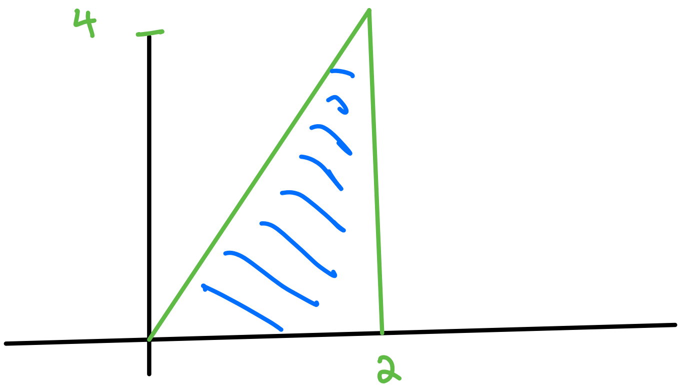
$$\Delta x = \frac{b-a}{N} = \frac{2-0}{N} = \frac{2}{N}$$

$$x_j = a + j\,\Delta x = 0 + j\left(\frac{2}{N}\right) = \frac{2j}{N}$$

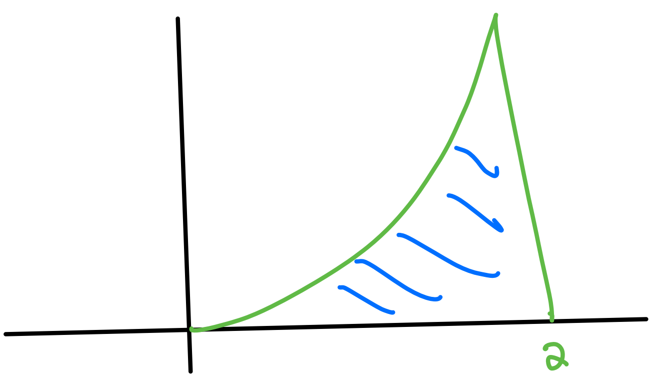
$$A \approx \sum_{j=1}^{N} f(x_j)\,\Delta x \approx \sum_{j=1}^{N} 2\left(\frac{2j}{N}\right)\cdot\frac{2}{N}$$

$$= \frac{8}{N^2}\sum_{j=1}^{N} j = \frac{8}{N^2}\,\frac{N(N+1)}{2}$$

$$\lim_{N\to\infty} \frac{8}{N^2}\,\frac{(N)(N+1)}{2} = \underline{\underline{4}}$$



$f(x) = x^2 \qquad [0,2] \qquad N\text{-SUB.}$



$$\Delta x = \frac{b-a}{N} = \frac{2-0}{N} = \frac{2}{N}$$

$$x_j = a + j\,\Delta x$$
$$= 0 + j\left(\frac{2}{N}\right) = \frac{2j}{N}$$

$$A \approx \sum_{j=1}^{N} f(x_j)\,\Delta x = \sum_{j=1}^{N} \left(\frac{2j}{N}\right)^2 \frac{2}{N}$$

$$= \sum_{j=1}^{N} \frac{4j^2}{N^2} \cdot \frac{2}{N} = \frac{8}{N^3} \sum_{j=1}^{N} j^2$$

$$= \frac{8}{N^3} \frac{N(N+1)(2N+1)}{6}$$

$$\lim_{N \to \infty} \frac{8}{N^3} \frac{N(N+1)(2N+1)}{6} = \frac{16}{6} = \frac{8}{3}$$

$f(x) = x^3$ $\quad [0,1] \quad$ N-SUB.

$$\Delta x = \frac{b-a}{N} = \frac{1-0}{N} = \frac{1}{N}$$

$$x_j = a + j\Delta x = 0 + j\left(\frac{1}{N}\right) = \frac{j}{N}$$

$$A \approx \sum_{j=1}^{N} f(x_j)\,\Delta x = \sum_{j=1}^{N} f\left(\frac{j}{N}\right) \cdot \frac{1}{N}$$

$$= \sum_{j=1}^{N} \left(\frac{j}{N}\right)^3 \cdot \frac{1}{N} = \sum_{j=1}^{N} \frac{j^3}{N^3} \cdot \frac{1}{N}$$

$$= \frac{1}{N^4} \sum_{j=1}^{N} j^3 = \frac{1}{N^4} \left(\frac{N(N+1)}{2}\right)^2$$

$$A = \lim_{N \to \infty} \frac{1}{N^4} \left(\frac{N(N+1)}{2}\right)^2 = \frac{1}{4}$$