

September 3, 2025

Welsh Rarebit Day

Today in History:

Treaty of Paris Signed (1783)

Viking 2 Lands on Mars (1976)

Number of the Day: 775

$$775 = 5 \times 5 \times 31$$

$$775 = 3^3 + 4^3 + 5^3 + 6^3 + 7^3.$$

Fun Fact:

People who use more emojis have more sex and get more dates. 😊

Quote of the Day:

"Life is either a daring adventure or nothing at all."

- Helen Keller

Today's Weather:

Sunny to partly cloudy, high 81°.

Math 121

Quiz #5

If $\lim_{x \rightarrow -4} f(x) = 3$ and $\lim_{x \rightarrow -4} g(x) = 4$,

find

$$\lim_{x \rightarrow -4} \frac{f(x) + 1}{3g(x) - 9}$$

$$= \frac{3 + 1}{12 - 9} = \frac{4}{3}$$

Pg 78 ✖ 9

$$\begin{aligned} \lim_{x \rightarrow -1} (3x^4 - 2x^3 + 4x) &= 3 + 2 - 4 \\ &= 1 \end{aligned}$$

mpod * 4
5

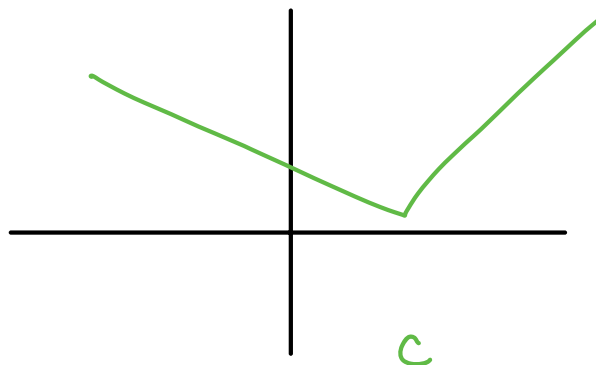
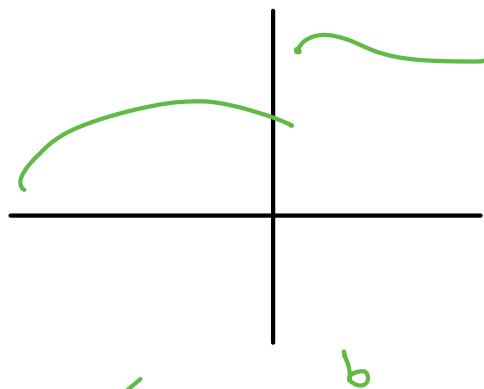
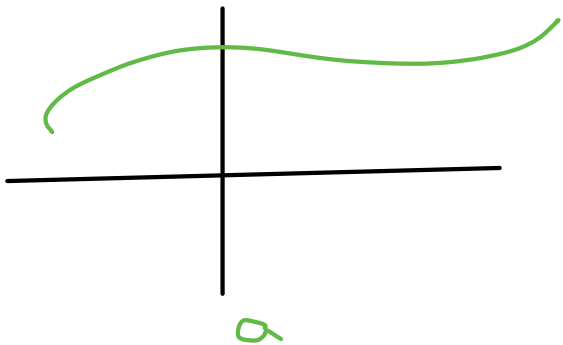
$$\lim_{x \rightarrow 0} \frac{\sin x}{x}$$

$$= 1$$

x	$\frac{\sin x}{x}$
.1	.998
.01	.9999
.0001	.99999...

CONTINUITY

WE SAY $f(x)$ IS CONT.



AT $x = c$ IF

$$① \lim_{x \rightarrow c} f(x) \text{ EXISTS}$$

$$② f(c) \text{ EXISTS}$$

$$\rightarrow ③ \lim_{x \rightarrow c} f(x) = f(c)$$

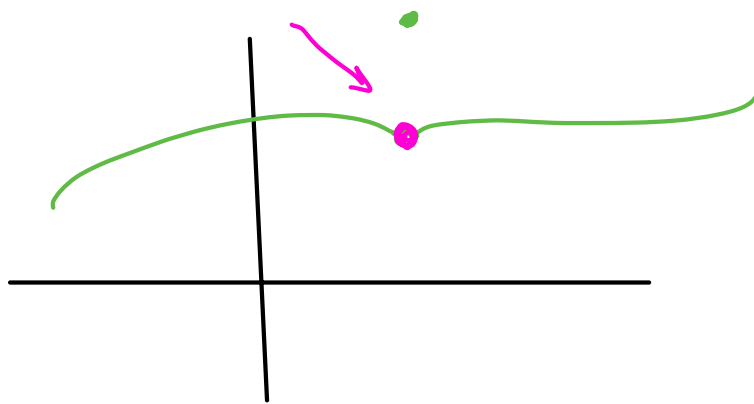
EXAMPLE

$$\rightarrow f(x) = 3 \quad \text{AT } x = 2$$

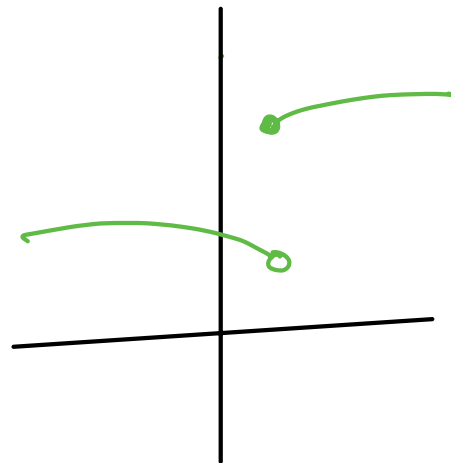
$$① \lim_{x \rightarrow 2} f(x) = 3$$

$$② f(2) = 3$$

$$③ \lim_{x \rightarrow 2} f(x) = f(2) \quad \checkmark \quad \text{YES.}$$



REMOVABLE



JUMP

ONE-SIDE CONT.

$$\text{LEFT CONT} \quad \lim_{x \rightarrow c^-} f(x) = f(c)$$

$$\text{RIGHT CONT} \quad \lim_{x \rightarrow c^+} f(x) = f(c)$$

EXAMPLE

$$f(x) = \sqrt{x}$$

IS IT CONT AT $x=0$?

$$\lim_{x \rightarrow 0} \sqrt{x}$$

~~$$\lim_{x \rightarrow 0^-} \sqrt{x}$$~~

$$\lim_{x \rightarrow 0^+} \sqrt{x}$$

RIGHT-CONT

CONT ON INTERVAL

(a, b) OPEN

$[a, b]$ CLOSED

$(a, b]$

$[a, b)$

WHICH FUNCTIONS ARE CONT.

$f(x), g(x)$ ARE CONT.

POLYNOMIALS

RATIONAL

$$\frac{p(x)}{q(x)}$$

$$q(x) \neq 0$$

SUM OF CONT. $f(x) \pm g(x)$

PROD. $f(x) \cdot g(x)$

QU.

$$\frac{f(x)}{g(x)}$$

$$g(x) \neq 0$$

$\sin x, \cos x, b^x (b > 0), \log_b x (x > 0)$

$$(f \circ g)(x)$$

EXAMPLES

$$\textcircled{1} \quad f(x) = x^2 + \cos x \quad \forall x$$

$$\textcircled{2} \quad f(x) = \frac{1}{x^2 - 1} \quad x \neq \pm 1$$

$$\textcircled{3} \quad f(x) = \frac{x^2 - 4}{x - 2} \quad x \neq 2$$

PIECE-WISE FUNCTIONS

$$f(x) = \begin{cases} x+1 & x \leq 0 \\ x^2+1 & x > 0 \end{cases}$$

$$f(-2) = -2+1 = -1$$

$$f(3) = 3^2+1 = 10$$

$$f(x) = \begin{cases} x^3 & x < 2 \\ ax^2 & x \geq 2 \end{cases}$$

WHAT VALUE OF a MAKES $f(x)$ CONT.?

$$f(2) = 4a \quad \lim_{x \rightarrow 2} f(x)$$

$$\lim_{x \rightarrow 2^-} f(x) = 8$$

$$\lim_{x \rightarrow 2^+} f(x) = 4a$$

$$8 = 4a$$

$$a = 2$$

FIND a AND b SO THAT

$$f(x) = \begin{cases} 2 & x < -1 \\ ax+b & -1 \leq x < 2 \\ -2 & x \geq 2 \end{cases}$$

CONT.

PROB. SPOTS ARE $x = -1$, $x = 2$

$$f(-1) = -a + b$$

$$f(2) = -2$$

$$\lim_{x \rightarrow -1^-} f(x) = 2$$

$$\lim_{x \rightarrow -1^+} f(x) = -a + b$$

$$\lim_{x \rightarrow 2^-} f(x) = 2a + b$$

$$\lim_{x \rightarrow 2^+} f(x) = -2$$

$$\begin{array}{r} 2(-a + b = 2) \\ 2a + b = -2 \\ \hline -2a + 2b = 4 \\ \hline 3b = 2 \end{array}$$

$$b = \frac{2}{3}$$

$$2a + \frac{2}{3} = -2$$

$$a = -\frac{4}{3}$$