December 1, 2025

Rosa Parks Day

Today in History:

Chunnel makes breakthrough (1990)

Congress decides outcome of presidential election (1824)

Number of the Day: 3048

3048 = 2 x 2 x 2 x 3 x 127.

The **3048**th prime divides the **3048**th Fibonacci number

Fun Fact:

In France, it is illegal to name your pig Napoleon.

Quote of the Day:

"I'm sorry, if you were right, I'd agree with you."

–Robin Williams

Today's Weather:

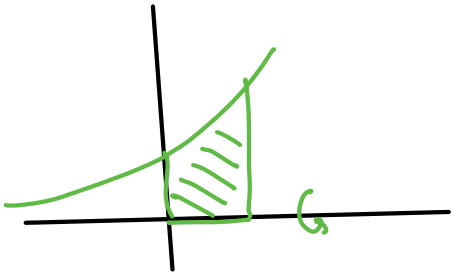
Cloudy skies, snow tonight. High of 32^{o}.

Math 121 - Quiz #45

Find the volume of the solid formed if the region bounded by

$y = e^x$ $\quad y = 0$ $\quad x = 0$ $\quad$ and $\quad x = 1$

is revolved about the x-axis.



$$V = \int_0^1 \pi (e^x)^2 \, dx$$

$$= \pi \int_0^1 e^{2x} \, dx$$

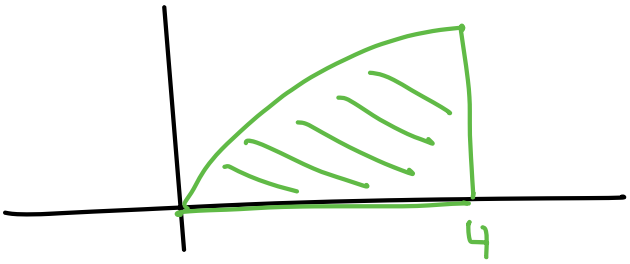
$$= \pi \left(\frac{e^{2x}}{2} \right) \Big|_0^1 = \frac{\pi}{2} (e^2 - 1)$$

Math 121 - Quiz #46

Find the volume of the solid formed if the region bounded by

$$y = \sqrt{x} \qquad y = 0 \qquad \text{and} \qquad x = 4$$

is revolved about the x-axis.

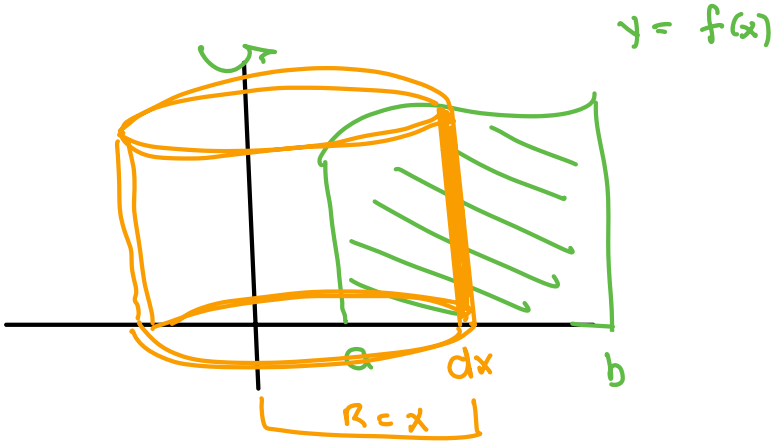


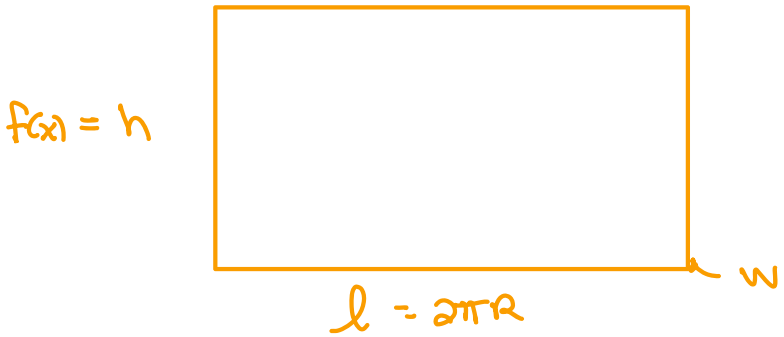
$$V = \int_0^4 \pi (\sqrt{x})^2 \, dx$$

$$= \pi \int_0^4 x \, dx$$

$$= \pi \left. \frac{x^2}{2} \right|_0^4 = 8\pi$$

VOLUME BY SHELLS



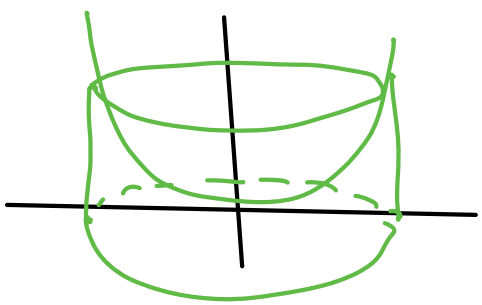


$$V = h \cdot l \cdot w$$

$$f(x) \cdot 2\pi x \, dx$$

$$\boxed{V = \int_a^b 2\pi x \, f(x) \, dx}$$

EXAMPLE 1 $\quad f(x) = x^2 \quad [0,2] \quad y\text{-AXIS}$

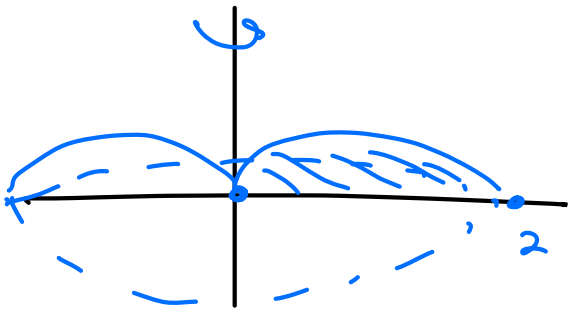


$$V = \int_0^2 2\pi x \, (x^2) \, dx$$

$$= 2\pi \int_0^2 x^3 \, dx$$

$$= 2\pi \left(\frac{x^4}{4} \right) \Big|_0^2 = 8\pi$$

$f(x) = 2x^2 - x^3$ $\quad y = 0$ $\quad y$-AXIS



$$0 = 2x^2 - x^3$$
$$= x^2(2 - x)$$
$$x = 0, 2$$

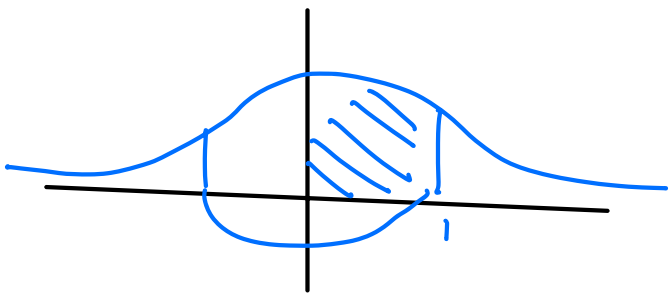
$$V = \int_0^2 2\pi x (2x^2 - x^3)\, dx$$

$$= 2\pi \int_0^2 2x^3 - x^4\, dx$$

$$= 2\pi \left[\frac{2x^4}{4} - \frac{x^5}{5} \right] \Big|_0^2$$

$$= \frac{16\pi}{5}$$

EXAMPLE 3 $\quad f(x) = e^{-x^2}$ $\quad [0,1]$ $\quad y$-AXIS



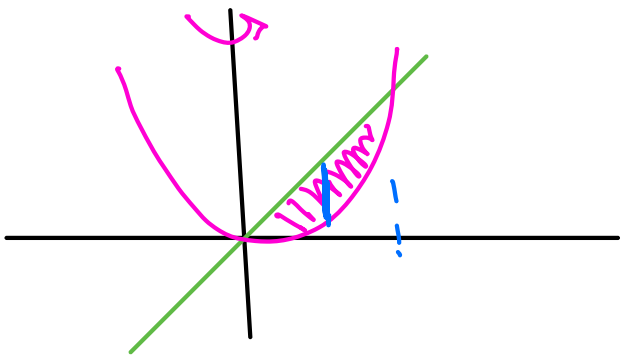
$$V = \int_0^1 2\pi x\, e^{-x^2}\, dx$$

$$u = -x^2$$
$$du = -2x\, dx$$
$$dx = \frac{du}{-2x}$$

$$= \int \cancel{2\pi x}\, e^{u}\, \frac{du}{\cancel{-2x}} \qquad = -\pi \int e^{u}\, du$$

$$= -\pi\, e^{u} = -\pi e^{-x^2}\Big|_0^1 = -\pi\left[e^{-1} - 1\right]$$

EXAMPLE 4 $\quad y = x \quad y = x^2 \quad$ ABOUT y-AXIS



$$V = \int_a^b 2\pi x\, f(x)\, dx$$

$f(x)$ IS HEIGHT

HEIGHT = TOP − BOTTOM

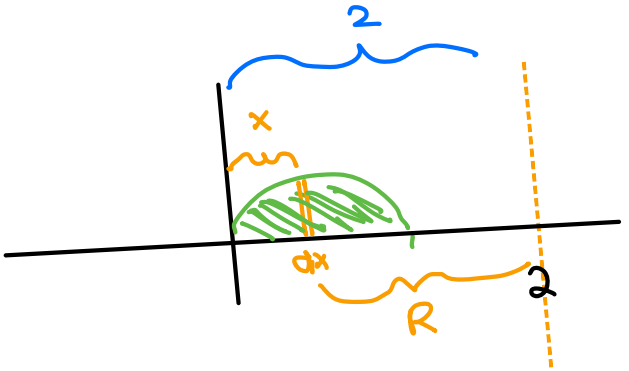
$$V = \int_0^1 2\pi x\left(x - x^2\right) dx$$

$$= 2\pi \int_0^1 x^2 - x^3\, dx = 2\pi\left[\frac{x^3}{3} - \frac{x^4}{4}\right]_0^1$$

$$= 2\pi\left[\frac{1}{3} - \frac{1}{4}\right] = \frac{\pi}{6}$$

EXAMPLE 5 $\quad y = x - x^2 \quad y = 0$

ABOUT $x = 2$



$$V = \int_a^b 2\pi \underline{x}\ \underline{f(x)}\, dx$$

$$= \int_a^b 2\pi\, R\, h\, dx$$

$R + x = 2 \qquad R = 2 - x$

$$V = \int_0^1 2\pi \underbrace{(2-x)}_{R}\underbrace{(x - x^2)}_{h}\, dx$$

$$= 2\pi \int_0^1 2x - 2x^2 - x^2 + x^3\, dx$$

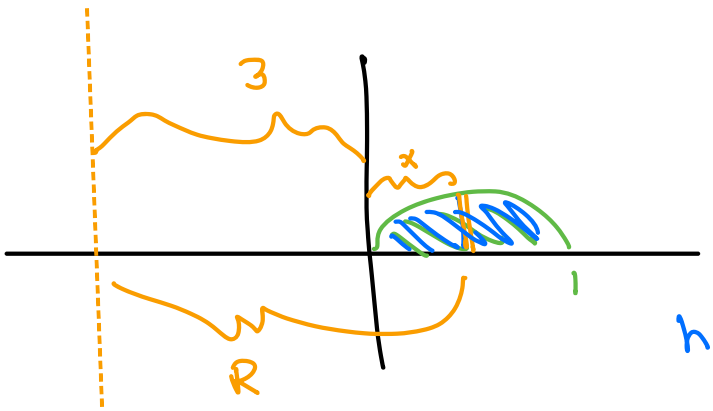
$$= 2\pi \int_0^1 2x - 3x^2 + x^3\, dx$$

$$= 2\pi \left[x^2 - x^3 + \frac{x^4}{4} \right]_0^1 = 2\pi \left[1 - 1 + \frac{1}{4} \right]$$

$$= \frac{\pi}{2}$$

EXAMPLE 6 $\qquad y = x - x^2 \qquad y = 0$

ABOUT $x = -3$



$$V = \int_a^b 2\pi R h\, dx$$

$$h = x - x^2$$

$3 + x = R$

$$V = \int_0^1 2\pi (x+3)(x - x^2)\, dx$$

$$= 2\pi \int_0^1 (x^2 - x^3 + 3x - 3x^2)\, dx$$

$$= 2\pi \int_0^1 -2x^2 - x^3 + 3x \, dx$$

$$= 2\pi \left[-\frac{2x^3}{3} - \frac{x^4}{4} + \frac{3x^2}{2} \right]_0^1$$

$$= 2\pi \left[-\frac{2}{3} - \frac{1}{4} + \frac{3}{2} \right]$$

RECAP

X-AXIS OR PAR. X-AXIS

$$V = \int_a^b \pi \left(OUT^2 - IN^2 \right) dx$$

Y-AXIS OR PAR Y-AXIS

$$V = \int_a^b 2\pi R h \, dx$$

R IS DIST TO AXIS

h IS HEIGHT OF SLICE