First Order Linear Differential Equations

Solve the following differential equations:

1. $2y' + 3y = e^{-x}$

2. $\frac{dy}{dx} - \frac{2x}{x^2+1}y = x$

3. $\frac{dy}{dx} + y \cot x = 1$

4. $\cos x \frac{dy}{dx} + y \sin x = 0$

5. $\frac{dy}{dx} - 3y = e^{3x} \sin x$

Answers

1. $y = e^{-x} + Ce^{-3x/2}$

2. $y = (x^2 + 1) \left[\frac{1}{2} \ln(x^2 + 1) + C\right]$

3. $y = -\cot x + \frac{C}{\sin x}$

4. $y = C \cos x$

5. $y = -e^{3x} [\cos x + C]$