Math 122 - #16
Logistic Equation

1. Find the solution of $y' = \frac{1}{2}y\left(1 - \frac{y}{4}\right)$

   a. with initial condition $y(0) = 1$

   b. with initial condition $y(0) = 6$

   c. with initial condition $y(0) = -1$

2. Find the solution of $y' = -\frac{1}{3}y\left(3 - y\right)$

   a. with initial condition $y(0) = 2$

   b. with initial condition $y(0) = -2$

   c. with initial condition $y(0) = 3.5$

3. 40 bears are released into a game refuge. After 5 years, the bear population is 104. The environment can support no more than 4000 bears. The growth rate of the population p is

$$p' = k\,p\left(1 - \frac{p}{4000}\right)$$

How many bears are there after 15 years?

4. In a class of 100 students, 10 students have heard the rumor that the next exam will be very hard. After 1 week, the number of student that have heard the rumor has increased to 20 students. Assuming the number of students that have heard the rumor follows the logistic differential equation, when will 80% of the class have heard the rumor.

Answers

1. **a.** $y = \dfrac{4}{1 + 3e^{-t/2}}$

   b. $y = \dfrac{4}{1 - \frac{1}{3}e^{-t/2}}$

   c. $y = \dfrac{4}{1 - 5e^{-t/2}}$

2. **a.** $y = \dfrac{6}{2 + e^t}$

   b. $y = \dfrac{6}{2 - 5e^t}$

   c. $y = \dfrac{21}{7 - e^t}$

3. 626 bear

4. 4.42 weeks