Math 121
Outline of Topics

I. Pre-Calculus

A. Functions

1. Domain
2. Range
3. Composition
4. Vertical Line Test

B. Inverse Functions

1. Domain
2. Range
3. Logs and Exponentials
4. Inverse Trig
5. Hyperbolic

II. Limits and Continuity

A. Limits

1. Computing limits by factoring
2. Limits at $\pm\infty$
3. Squeeze Law
4. One-sided Limits
5. Definition of a limit using $\epsilon - \delta$.
6. $\lim_{x \to 0} \dfrac{\sin x}{x}$

B. Continuous Functions

1. $f(a)$ must be defined
2. $\lim_{x \to a} f(x)$ must exist
3. $\lim_{x \to a} f(x) = f(a)$
4. Graphical interpretation
5. Examples of discontinuity

III. Differentiation

A. Definition of Derivative

1. $f'(x) = \lim_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$
2. $f'(a) = \lim_{x \to a} \dfrac{f(x) - f(a)}{x - a}$
3. The derivative is the slope of the tangent line.
4. Graphical interpretation

B. Sums, difference, products, quotient and the power rule

1. $\dfrac{d}{dx} cf(x) = cf'(x)$
2. $\dfrac{d}{dx}(f(x) \pm g(x)) = f'(x) \pm g'(x)$
3. $\dfrac{d}{dx}(f(x)g(x)) = f(x)g'(x) + g(x)f'(x)$
4. $\dfrac{d}{dx}\left[\dfrac{f(x)}{g(x)}\right] = \dfrac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$
5. $\dfrac{d}{dx} x^n = nx^{n-1}$

C. Trig Functions

1. $\frac{d}{dx}\sin x = \cos x$
2. $\frac{d}{dx}\cos x = -\sin x$
3. $\frac{d}{dx}\tan x = \sec^2 x$
4. $\frac{d}{dx}\sec x = \sec x \tan x$
5. $\frac{d}{dx}\cot x = -\csc^2 x$
6. $\frac{d}{dx}\csc x = -\csc x \cot x$

D. Chain Rule

1. $\frac{d}{dx}(f \circ g)(x) = (f' \circ g)(x)\ g'(x)$

E. Inverse Functions

1. If $g(x) = f^{-1}(x)$ then $g'(b) = \frac{1}{f'(g(b))}$
2. $\frac{d}{dx}\arctan x = \frac{1}{1+x^2}$
3. $\frac{d}{dx}\arcsin x = \frac{1}{\sqrt{1-x^2}}$
4. $\frac{d}{dx}\text{arcsec } x = \frac{1}{|x|\sqrt{x^2-1}}$

F. Logs and Exponentials

1. $\frac{d}{dx}\ln x = \frac{1}{x}$
2. $\frac{d}{dx}e^x = e^x$
3. $\frac{d}{dx}\log_a x = \frac{1}{x \ln a}$
4. $\frac{d}{dx}a^x = a^x \ln a$

5. $\frac{d}{dx}\sinh x = \cosh x$
6. $\frac{d}{dx}\cosh x = \sinh x$
7. $\frac{d}{dx}\tanh x = \operatorname{sech}^2 x$

G. Implicit Differentiation

IV. Applications of Differentiation

A. Related Rates

1. Read the problem
2. Draw a picture
3. Write an equation
4. Implicitly differentiate (differentiate everything)
5. Plug in and solve
6. Answer the question

B. Linear Approximations

1. $\Delta y \approx f'(x)\Delta x$
2. $f(x + \Delta x) \approx f(x) + f'(x)\Delta x$
3. $f(x) \approx L(x) = f(a) + f'(a)(x - a)$

C. Rolle's Theorem

1. $f(x)$ is continuous and differentiable
2. $f(a) = f(b) = 0$
3. $a < c < b$
4. $f'(c) = 0$

D. Mean-value Theorem

1. $f(x)$ is continuous and differentiable
2. $a < c < b$
3. $f'(c) = \frac{f(b)-f(a)}{b-a}$

E. Graphing

1. Domain range, x-intercepts, y-intercepts
2. First derivative: increasing, decreasing, critical points
3. Second derivative: Concave up, concave down, inflection points
4. Vertical and horizontal asymptotes

F. l'Hospital's Rule

1. $\frac{0}{0}$ and $\frac{\infty}{\infty}$
2. $0 \cdot \infty, 0^0, \infty^0, \infty - \infty$ or 1^∞

G. Max-Mins

1. Read the problem
2. Draw a picture
3. Write an equation
4. Make the equation have just one independent variable
5. Differentiate
6. Set equal to zero and solve
7. Determine if max or min (and check endpoints if possible)
8. Answer the question

H. Newton's Method

1. Solve for where $f(x) = 0$
2. $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

V. Integration

A. Riemann Sums

1. $\Delta x = \frac{b-a}{n} \qquad x_k = a + k\Delta x$
2. $\lim_{n\to\infty} \sum_{k=1}^{n} f(x_k)\Delta x$

B. Integration Formulae

1. $\int f(x) \pm g(x)\ dx = \int f(x)\ dx \pm \int g(x)\ dx$
2. $\int cf(x)\ dx = c\int f(x)\ dx$
3. $\int x^n\ dx = \frac{x^{n+1}}{n+1} + C$ for $n \neq -1$
4. $\int \sin x\ dx = -\cos x + C$
5. $\int \cos x\ dx = \sin x + C$
6. $\int \sec^2 x\ dx = \tan x + C$
7. $\int \frac{1}{x}\ dx = \ln|x| + C$
8. $\int e^x\ dx = e^x + C$

C. Substitution

1. $\int \left[f(u) \frac{du}{dx} \right] \, dx = \int f(u) \, du$
2. $u = g(x)$
3. $\frac{du}{dx} = g'(x)$

D. First Fundamental Theorem of Calculus

1. $\int_a^b f(x) \, dx = F(b) - F(a)$
2. $\frac{d}{dx} F(x) = f(x)$

E. Second Fundamental Theorem of Calculus

1. $\frac{d}{dx} \left[\int_a^x f(t) \, dx \right] = f(x)$
2. $\frac{d}{dx} \left[\int_a^{g(t)} f(t) \, dx \right] = g'(x) f(g(x))$

VI. Applications of Integration

A. Area

1. Area under a curve.
2. Area between two curves.

B. Average value

1. $f_{AVE} = \frac{1}{b-a} \int_a^b f(x) \, dx$

C. Volume

1. $V = \int_a^b \pi \left(f(x)\right)^2 \, dx$ slices (about x-axis).

2. $V = \int_a^b 2\pi x \; f(x) \, dx$ shells (about y-axis).

D. Work

1. $W = \int_a^b F(x) \, dx$