Math 223 Test 3

July 17, 2024

EF:

Name KEY

1 - 2	
3 - 4	
5 - 6	
7 - 8	
9 - 10	
Total	

Directions:

1. No books, notes, or adopting puppies from gas stations. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.

2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.

3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.

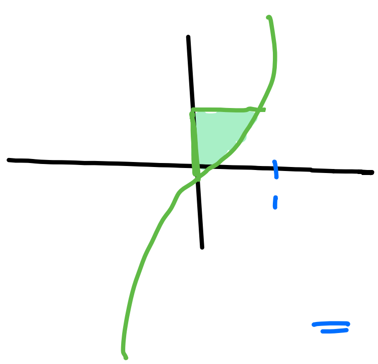
4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

1. (10 points) Compute $\int_{-1}^{1}\int_{x}^{2x} e^{x+y} dy\, dx$.

$$= \int_{-1}^{1} e^{x+y}\Big|_{x}^{2x} dx = \int_{-1}^{1} e^{3x} - e^{2x}\, dx$$

$$= \frac{e^{3x}}{3} - \frac{e^{2x}}{2}\Big|_{-1}^{1} = \left(\frac{e^{3}}{3} - \frac{e^{2}}{2}\right) - \left(\frac{e^{-3}}{3} - \frac{e^{-2}}{2}\right)$$

2. (10 points) Compute $\iint_R xy\, dA$, where R is the region bounded by $y = x^3, y = 1$, and $x = 0$

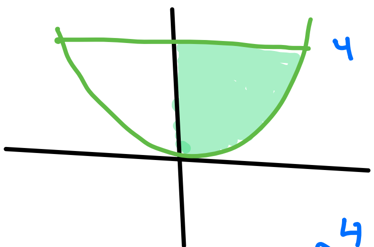


$$= \int_{0}^{1}\int_{x^3}^{1} xy\, dy\, dx$$

$$= \int_{0}^{1} \frac{xy^2}{2}\Big|_{x^3}^{1} dx = \int_{0}^{1}\left(\frac{x}{2} - \frac{x^7}{2}\right) dx$$

$$= \frac{x^2}{4} - \frac{x^8}{16}\Big|_{0}^{1} = \frac{1}{4} - \frac{1}{16} = \frac{3}{16}$$

3. (10 points) Compute $\displaystyle\int_0^2 \int_{x^2}^4 x\cos(y^2)\; dy\; dx$.



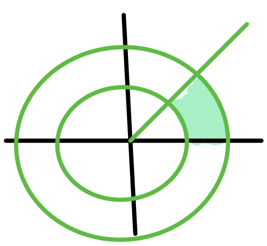
$$= \int_0^4 \int_0^{\sqrt{y}} x\cos(y^2)\; dx\; dy$$

$$= \int_0^4 \frac{x^2}{2}\cos(y^2)\Big|_0^{\sqrt{y}}\; dy = \int_0^4 \frac{y}{2}\cos(y^2)\, dy$$

$$= \frac{\sin(y^2)}{4}\Big|_0^4 = \frac{1}{4}\big(\sin(16)\big)$$

4. (10 points) Compute $\displaystyle\iint_R (x-y)\; dA$. Where R is the region in the first quadrant bounded by:

$$x^2+y^2=4, \qquad x^2+y^2=16, \qquad y=0, \qquad \text{and} \qquad y=x.$$



$$\int_0^{\frac{\pi}{4}} \int_2^4 (R\cos\theta - R\sin\theta)\; R\; dR\; d\theta$$

$$\int_0^{\frac{\pi}{4}} \frac{R^3}{3}\Big|_2^4 (\cos\theta - \sin\theta)\; d\theta$$

$$\left(\frac{64}{3} - \frac{8}{3}\right)\left(\sin\theta + \cos\theta\right)\Big|_0^{\frac{\pi}{4}} = \frac{56}{3}\left(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2} - 1\right)$$

$$= \frac{56}{3}\left(\sqrt{2} - 1\right)$$

5. (10 points) Compute $\displaystyle\int_0^1 \int_0^2 \int_0^{1-x-y} y \, dz \, dx \, dy$.

$$= \int_0^1 \int_0^2 yz \Big|_0^{1-x-y} dx \, dy$$

$$= \int_0^1 \int_0^2 (y - yx - y^2) \, dx \, dy$$

$$= \int_0^1 yx - \frac{yx^2}{2} - y^2 x \Big|_0^2 \, dy$$

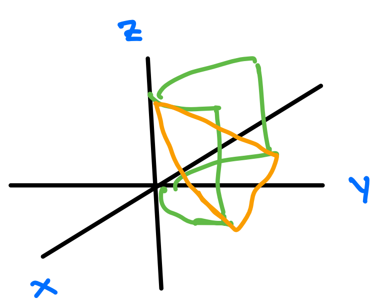
$$= \int_0^1 2y - 2y - 2y^2 \, dy = -\frac{2y^3}{3} \Big|_0^1$$

$$= -\frac{2}{3}$$

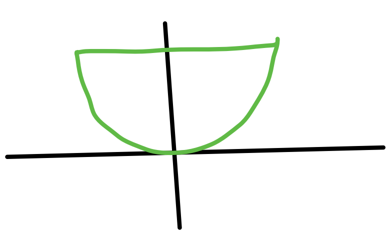
6. (10 points) Express

$$\iiint_B f(x, y, z) \, dV$$

and where B is the region bounded by $y = x^2$ and the planes $z = 0$ and $z = 4 - y$ as an iterated integral:

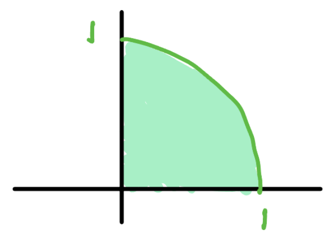


$$\int_{\boxed{-2}}^{\boxed{2}} \int_{\boxed{x^2}}^{\boxed{4}} \int_{\boxed{0}}^{\boxed{4-y}} f(x, y, z) \, dz \, dy \, dx$$



7. (10 points) Compute $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{2xy} (x^2+y^2)\, dz\, dy\, dx$.

$$= \int_0^1 \int_0^{\sqrt{1-x^2}} z\,(x^2+y^2)\Big|_0^{2xy}\, dy\, dx$$



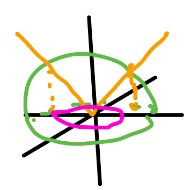
$$= \int_0^1 \int_0^{\sqrt{1-x^2}} 2xy\,(x^2+y^2)\, dx\, dy$$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 2R^2 \cos\theta \sin\theta\; R^2\; (R)\; dR\, d\theta$$

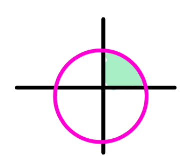
$$= \int_0^{\frac{\pi}{2}} \frac{R^6}{3}\Big|_0^1 \cos\theta\sin\theta\, d\theta = \frac{1}{3}\, \frac{\sin^2\theta}{2}\Big|_0^{\frac{\pi}{2}} = \frac{1}{6}$$

8. (10 points) Compute $\int_0^3 \int_0^{\sqrt{9-y^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{18-x^2-y^2}} \sqrt{x^2+y^2+z^2}\, dz\, dx\, dy$.

$$= \int_0^{\frac{\pi}{4}} \int_0^{\frac{\pi}{2}} \int_0^{3\sqrt{2}} \rho\; \rho^2 \sin\phi\; d\rho\, d\theta\, d\phi$$



$$\sqrt{x^2+y^2} = \sqrt{18-x^2-y^2} \qquad x^2+y^2 = 18-x^2-y^2$$



$$x^2+y^2 = 9$$

$$\int_0^{3\sqrt{2}} \rho^3\, d\rho \int_0^{\frac{\pi}{2}} d\theta \int_0^{\frac{\pi}{4}} \sin\phi\, d\phi$$

$$\left(\frac{\rho^4}{4}\Big|_0^{3\sqrt{2}}\right)\left(\frac{\pi}{2}\right)\left(-\cos\theta\Big|_0^{\frac{\pi}{4}}\right) = 81\left(\frac{\pi}{2}\right)\left(1-\frac{\sqrt{2}}{2}\right)$$

9. (10 points) Find the Jacobian $\dfrac{\partial(x,y)}{\partial(u,v)}$ for $u = 2x - 3y$ and $v = 4x - 5y$.

$$\frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} 2 & -3 \\ 4 & -5 \end{vmatrix} = -10 + 12 = 2$$

$$\frac{\partial(x,y)}{\partial(u,v)} = \frac{1}{2}$$

10. (10 points) Compute $\displaystyle\iint_R \frac{2x-3y}{4x-5y}\, dA$, where R is the region bounded by $2x - 3y = 0, 2x - 3y = 5, 4x - 5y = 1$ and $4x - 5y = 8$.

$$= \int_1^8 \int_0^5 \frac{u}{v} \, \frac{1}{2} \, du \, dv = \int_1^8 \frac{u^2}{4} \frac{1}{v} \Big|_0^5 \, dv$$

$$= \frac{25}{4} \int_1^8 \frac{1}{v} \, dv = \frac{25}{4} \ln v \Big|_1^{\infty} = \frac{25}{4} \ln 8$$