

Math 223 Test 3

July 15, 2025

EF:

Name KEY

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8	
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10	
Total	

Directions:

1. No books, notes, or over 1700 incoming first year students. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.
2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.
3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.
4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

1. (10 points) Compute

$$\int_0^1 \int_x^{\sqrt{x}} xy \, dy \, dx$$

$$= \int_0^1 x \left. \frac{y^2}{2} \right|_x^{\sqrt{x}} dx = \int_0^1 x \left(\frac{x}{2} - \frac{x^2}{2} \right) dx$$

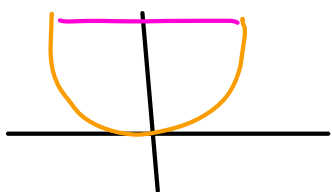
$$= \int_0^1 \left(\frac{x^2}{2} - \frac{x^3}{2} \right) dx = \frac{x^3}{6} - \frac{x^4}{8} \Big|_0^1 =$$

$$\frac{1}{6} - \frac{1}{8} = \frac{2}{48} = \frac{1}{24}$$

2. (10 points) Compute

$$\iint_R x^2 \, dA$$

where R bounded by $y = x^2$ and $y = 4$



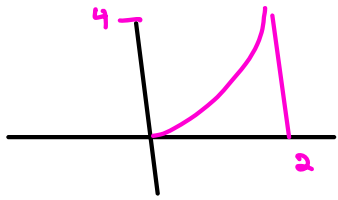
$$= \int_{-2}^2 \int_{x^2}^4 x^2 \, dy \, dx$$

$$= \int_{-2}^2 x^2 y \Big|_{x^2}^4 dx = \int_{-2}^2 x^2 (4 - x^2) dx$$

$$= \int_{-2}^2 4x^2 - x^4 dx = \frac{4x^3}{3} - \frac{x^5}{5} \Big|_{-2}^2$$

$$= \left(\frac{32}{3} - \frac{32}{5} \right) - \left(-\frac{32}{3} + \frac{32}{5} \right) = \frac{128}{15}$$

3. (10 points) Compute $\int_0^4 \int_{\sqrt{y}}^2 \cos(1+x^3) dx dy$.



$$\int_0^2 \int_0^{x^2} \cos(1+x^3) dy dx$$

$$= \int_0^2 x^2 \cos(1+x^3) dx$$

$$= \frac{\sin(1+x^3)}{3} \Big|_0^2 = \frac{1}{3} (\sin(9) - \sin(1))$$

4. (10 points) Compute

$$\iint_R (x^2 + y^2) dA$$

where R is the region inside the circle $x^2 + y^2 = 1$, $x \geq 0$, between the lines $y = 0$, and $y = x$.

$$= \int_0^{\frac{\pi}{4}} \int_0^1 r^2 r dr d\theta = \int_0^{\frac{\pi}{4}} \frac{r^4}{4} \Big|_0^1 d\theta$$

$$= \int_0^{\frac{\pi}{4}} \frac{1}{4} d\theta = \frac{\pi}{16}$$

5. (10 points) Compute

$$\iiint_B x^2 e^{y+3z} dV$$

where B is the box with $1 \leq x \leq 4$, $0 \leq y \leq 3$, $2 \leq z \leq 6$.

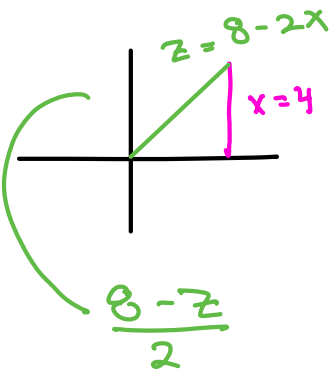
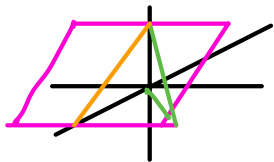
$$\begin{aligned} &= \left(\int_1^4 x^2 dx \right) \left(\int_0^3 e^y dy \right) \left(\int_2^6 e^{3z} dz \right) \\ &= \left(\frac{x^3}{3} \Big|_1^4 \right) \left(e^y \Big|_0^3 \right) \left(\frac{e^{3z}}{3} \Big|_2^6 \right) \\ &= (21)(e^3 - 1) \left(\frac{e^{18} - e^6}{3} \right) \end{aligned}$$

6. (10 points) Rewrite the integral

$$\int_0^4 \int_0^{3x} \int_0^{8-2x} f(x, y, z) dz dy dx$$

in the order $dy dx dz$.

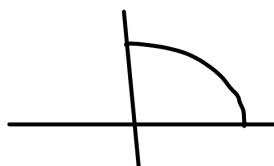
$$\begin{aligned} 0 &\leq z \leq 8 - 2x \\ 0 &\leq y \leq 3x \\ 0 &\leq x \leq 4 \end{aligned}$$



8	$\frac{8-z}{2}$	3x	$f(x, y, z) dy dx dz$
$\int$	$\int$	$\int$	
0	0	0	

7. (10 points) Compute

$$\int_0^1 \int_0^{\sqrt{1-x^2}} \int_1^{2-\sqrt{x^2+y^2}} \frac{xy}{(x^2+y^2)\sqrt{x^2+y^2}} dz dy dx$$



$$= \int_0^{\frac{\pi}{2}} \int_0^1 \int_1^{2-R} \frac{\cancel{R} \cos \theta \cancel{R} \sin \theta}{\cancel{R^2}} R dz dR d\theta$$

$$= \int_0^{\frac{\pi}{2}} \int_0^1 (2-R-1) \cos \theta \sin \theta dR d\theta$$

$$= \int_0^{\frac{\pi}{2}} \left(R - \frac{R^2}{2} \right) \cos \theta \sin \theta d\theta = \frac{1}{2} \frac{\sin^2 \theta}{2} \bigg|_0^{\frac{\pi}{2}}$$

$$= \frac{1}{4}$$

8. (10 points) Compute

$$\iiint_B z^2 dV$$

where B is the region inside the sphere $x^2 + y^2 + z^2 = 9$ above the cone $z = \sqrt{x^2 + y^2}$.

$$= \int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^3 \rho^2 \cos^2 \varphi \rho^2 \sin \varphi d\rho d\varphi d\theta$$

$$= \int_0^{2\pi} d\theta \int_0^{\frac{\pi}{4}} \cos^2 \varphi \sin \varphi \int_0^3 \rho^4 d\rho$$

$$= (2\pi) \left(-\frac{\cos^3 \varphi}{3} \bigg|_0^{\frac{\pi}{4}} \right) \left(\frac{\rho^5}{5} \bigg|_0^3 \right)$$

$$= (2\pi) \left(-\frac{\left(\frac{\sqrt{2}}{2}\right)^3}{3} + 1 \right) \left(\frac{243}{5} \right)$$

9. (10 points) Find the Jacobian $\frac{\partial(x, y)}{\partial(u, v)}$ for $u = x - y$ and $v = xy$

$$\frac{\partial(u, v)}{\partial(x, y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix} = \begin{vmatrix} 1 & -1 \\ y & x \end{vmatrix} = x + y$$

$$\frac{\partial(x, y)}{\partial(u, v)} = \frac{1}{x + y}$$

10. (10 points) Compute the value of

$$\iint_R (x^2 - y^2) dA$$

where R is the region enclosed by $x - y = 0$, $x - y = 1$, $xy = 1$ and $xy = 5$.

$$= \int_1^5 \int_0^1 \cancel{(x^2 - y^2)}^{\cancel{x-y}} \left(\frac{1}{\cancel{x-y}} \right) du dv$$

$$= \int_1^5 \int_0^1 u du dv = \int_1^5 \frac{u^2}{2} \Big|_0^1 dv$$

$$= \int_1^5 \frac{1}{2} dv = \frac{1}{2} (5 - 1) = 2$$

FORMULA PAGE

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$

$$\cos^2 x = \frac{1 + \cos 2x}{2}$$

$$\sin 2x = 2 \sin x \cos x$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \sec^2 x$$

$$(\sec x)' = \sec x \tan x$$

$$(\csc x)' = -\csc x \cot x$$

$$(\cot x)' = -\csc^2 x$$

$$(e^x)' = e^x$$

$$(\ln x)' = \frac{1}{x}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arcsec} x)' = \frac{1}{|x|\sqrt{x^2-1}}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$f(c) = \frac{1}{b-a} \int_a^b f(x) dx$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C$$

$$\int \sec^3 x \, dx = \frac{1}{2} [\sec x \tan x + \ln |\sec x + \tan x|] + C$$

$$\int \csc x \, dx = \ln |\csc x - \cot x| + C$$

$$1 + 1 = 2$$