Math 223
Test 3 - Review 1

1. $\int_0^{\pi} \int_0^{\cos y} x \sin y \; dx \; dy$

2. $\int_{-1}^{2} \int_0^{1-x} (4-y) \; dy \; dx$

3. Evaluate the integral $\iint_R (x^2 + 4y) \; dA$ where R is the region bounded by the graphs of $y = 2x$ and $y = x^2$.

4. $\int_0^1 \int_{2x}^2 e^{y^2} \; dy \; dx.$

5. $\int_0^1 \int_x^1 \cos y^2 \; dy \; dx.$

6. Use a double integral to find the volume of the solid in the first octant bounded above by the plane $x + y + z = 4$ and below by the rectangle $0 \leq x \leq 1, 0 \leq y \leq 2$.

7. Find the volume of the solid that lies under the paraboloid $z = x^2 + y^2$ and above the region D in the xy-plane bounded by the line $y = 2x$ and the parabola $y = x^2$.

8. Evaluate the integral $\iint_R \sqrt{x^2 + y^2} \; dA$ where R is the region in the xy-plane enclosed by $x^2 + y^2 = 9$.

9. Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} e^{-(x^2+y^2)} \; dy \; dx$ using polar coordinates.

10. Evaluate the integral $\iint_R r \sin\theta \; dA$ where R is the region above the x-axis bounded by the semicircle $r = 2, 0 \leq \theta \leq \pi$.

11. Evaluate the integral $\iiint_T xyz^2 \; dV$ where T is the region $0 \leq x \leq 1, -1 \leq y \leq 2$, $0 \leq z \leq 3$.

12. Evaluate the integral $\iiint_T z \, dV$ where T is the region is the solid tetrahedron bounded by the four planes $x = 0, y = 0, z = 0$ and $x + y + z = 1$.

13. Use a triple integral to find the volume of the tetrahedron T bounded by the planes $x + 2y + z = 2, x = 2y, x = 0$, and $z = 0$.

14. Evaluate the integral $\iiint_T \sqrt{x^2 + y^2} \, dV$ where T is the region bounded by the cylinder $x^2 + y^2 = 1$, below the plane $z = 4$, and above the paraboloid $z = 1 - x^2 - y^2$.

15. $\int_{-2}^{2} \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^{2} (x^2 + y^2) \, dz \, dy \, dx$ (Hint: Use cylindrical coordinates).

16. Evaluate the integral $\iiint_T e^{(x^2+y^2+z^2)^{3/2}} \, dV$ where T is the unit ball $x^2 + y^2 + z^2 \leq 1$.

17. Find the Jacobian $\dfrac{\partial(x, y)}{\partial(u, v)}$ for $x = \sin u + \cos v, y = -\cos u + \sin v$

18. $\iint_R \dfrac{x - y}{x + y} \, dA$ where R is the region enclosed by $x - y = 0, x - y = 1, x + y = 1$ and $x + y = 3$.

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Answers

1. $\frac{1}{3}$

2. $\frac{9}{2}$

3. $\frac{152}{15}$

4. $\frac{1}{4}(e^4 - 1)$

5. $\frac{1}{2}\sin 1$

6. 5

7. $\frac{216}{35}$

8. 18π

9. $\frac{\pi(e-1)}{4e}$

10. $\frac{16}{3}$

11. $\frac{27}{4}$

12. $\frac{1}{24}$

13. $\frac{1}{3}$

14. $\frac{12\pi}{5}$

15. $\frac{16\pi}{5}$

16. $\frac{4\pi}{3}(e-1)$

17. $\cos(u-v)$

18. $\frac{1}{4}\ln 3$