SI:

Math 223 Test 2

EF:

July 2, 2025

Name______________________________

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Total	

Directions:

1. No books, notes, or spending 56 million dollars on your wedding. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.

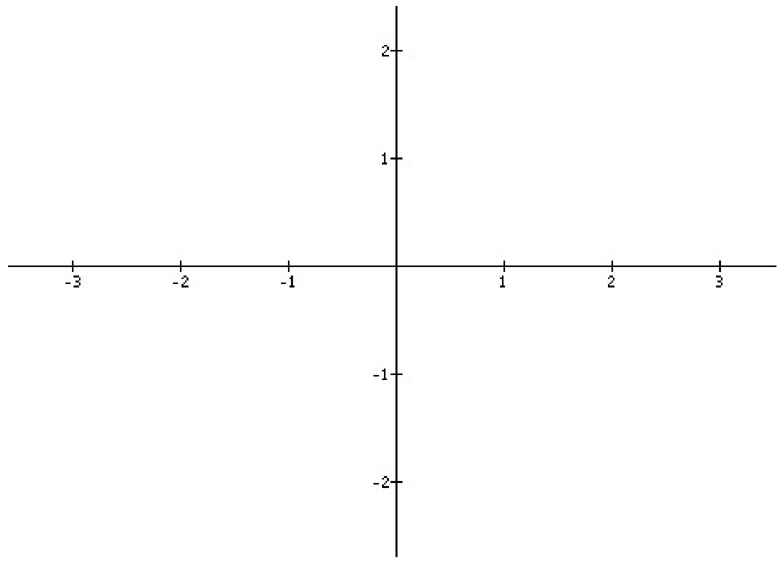
2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.

3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.

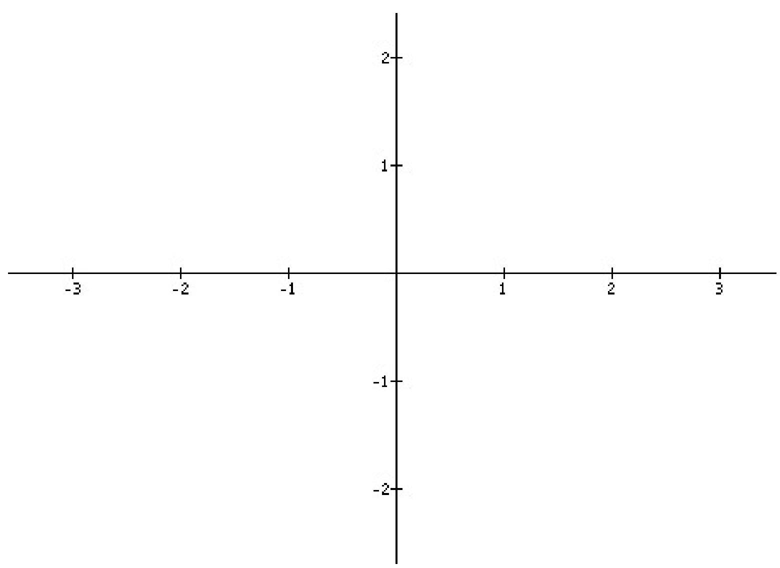
4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

1. (10 points) Find and sketch the domain of each of the following functions:

   (a) $f(x, y) = \dfrac{x}{\sqrt{4 - x^2 - y^2}}$



   (b) $f(x, y) = \dfrac{e^{xy}}{y^2 - x^2}$



2. (5 points) Find (or show it does not exist) $\displaystyle\lim_{(x,y)\to(2,2)} \frac{x^2+xy-4x}{\sqrt{x+y}-2}$

3. (10 points) For $f(x,y) = y^3 \ln x + x^2 e^y$

(a) $f_x(1,2)$

(b) $f_y(1,2)$

(c) $f_{xx}(1,2)$

(d) $f_{xy}(1,2)$

(e) $f_{yy}(1,2)$

(f) $f_{yx}(1,2)$

4. (10 points)

(a) Find the equation of the tangent plane to

$$z = x^2\sqrt{y} + y^2\sqrt{x}$$

at the point (4, 1).

(b) Estimate $f(2.02, 1.01, -0.03)$ assuming that

$$f(2,1,0) = -3 \qquad f_x(2,1,0) = -2, \qquad f_y(2,1,0) = 4, \qquad f_z(2,1,0) = 2$$

5. (10 points) Let $z = f(x, y)$ with $x = g(r, \theta)$ and $y = h(r, \theta)$, and $g(1, \pi/2) = -1$ and $h(1, \pi/2) = 1$. Also, you are given:

$$\frac{\partial f}{\partial x}(-1,1) = 2, \qquad \frac{\partial f}{\partial x}(1,\pi/2) = 3, \qquad \frac{\partial f}{\partial y}(-1,1) = 4, \qquad \frac{\partial f}{\partial y}(1,\pi/2) = 5$$

$$\frac{\partial x}{\partial r}(1,\pi/2) = 6, \qquad \frac{\partial x}{\partial \theta}(1,\pi/2) = 7, \qquad \frac{\partial y}{\partial r}(1,\pi/2) = 8, \qquad \frac{\partial y}{\partial \theta}(1,\pi/2) = 9$$

(a) Find $\frac{\partial f}{\partial r}$ at $r = 1$ and $\theta = \pi/2$

(b) Find $\frac{\partial f}{\partial \theta}$ at $r = 1$ and $\theta = \pi/2$

6. (10 points) If $F(x, y, z) = x^2 + y^2 - 2z^2 + 12x - 8z - 4 = 0$, find at $P(1, 1, 1)$

(a) $\frac{\partial z}{\partial x}$

(b) $\frac{\partial z}{\partial y}$

7. (15 points)

(a) Find a vector perpendicular to the surface

$$x^2 + y^2 + z^2 + x^2y^2 + x^2z^2 + y^2z^2 - z = 13$$

at the point $P = (1, 1, 2)$

(b) Find the equation of the tangent plane to the surface

$$x^2 + y^2 + z^2 + x^2y^2 + x^2z^2 + y^2z^2 - z = 13$$

at the point $P = (1, 1, 2)$

(c) Find the directional derivative of

$$f(x, y, z) = x^2 + y^2 + z^2 + x^2y^2 + x^2z^2 + y^2z^2 - z$$

at $P = (1, 1, 2)$ in the direction $\overrightarrow{v} = \langle -1, 3, 2 \rangle$

8. (15 points) Let $f(x, y) = 2x^3 + 6xy^2 - 3y^3 - 150x$. Find all critical points of $f(x, y)$ and determine whether they are local maxima, local minima, or saddle points.

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9. (15 points) Find the maximum and minimum of $f(x, y) = xy$ subject to the constraint

$$4x^2 + 9y^2 = 36$$

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FORMULA PAGE

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin 2x = 2\sin x\cos x$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \sec^2 x$$

$$(\sec x)' = \sec x\tan x$$

$$(\csc x)' = -\csc x\cot x$$

$$(\cot x)' = -\csc^2 x$$

$$(e^x)' = e^x$$

$$(\ln x)' = \frac{1}{x}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arcsec} x)' = \frac{1}{|x|\sqrt{x^2-1}}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$f(c) = \frac{1}{b-a}\int\limits_a^b f(x)dx$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x\,dx = \frac{1}{2}\left[\sec x\tan x + \ln|\sec x + \tan x|\right] + C$$

$$\int \csc x\,dx = \ln|\csc x - \cot x| + C$$

$$1 + 1 = 2$$