SI: | | EF: | |

Math 223 Test 1

June 16, 2025

Name______________________________

1	
2	
3	
4	
5	
6	
Total	

Directions:

1. No books, notes, or getting 6 outs with only 9 pitches. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.

2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.

3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.

4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

1. (15 points) For vectors $\overrightarrow{a} = \langle 2, -1, 3\rangle$ and $\overrightarrow{b} = \langle 4, 0, -1\rangle$ find:

(a) $2\overrightarrow{a} + \overrightarrow{b}$

(b) $||2\overrightarrow{a} + \overrightarrow{b}||$

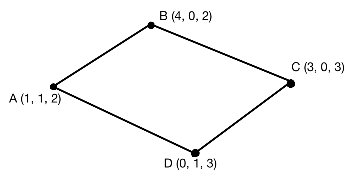
(c) $\overrightarrow{a} \cdot \overrightarrow{b}$

(d) $\vec{a} \times \vec{b}$

(e) $\cos\theta$, where θ is the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$

2. (15 points)

(a) Find the area of the parallelogram $ABCD$ for the points $A(1,1,2), B(4,0,2), C(3,0,3)$, and $D(0,1,3)$.



(b) Find the equation of the line which passes through the point $(1,-2,3)$ and is parallel to the line containing the points $(3,2,0)$ and $(4,7,2)$.

(c) Find the equation of the plane contains the point $(4,-5,-1)$ and is perpendicular to the line

$$x = 3 - t, \qquad y = 4 + 4t, \qquad z = 9$$

3. (15 points)

(a) Find the equation of the plane whose points are equidistant from $A(2, 3, -1)$ and $B(6, -1, 5)$

(b) Find the equation of the line of intersection of the two planes

$$x + y + z = 2 \qquad x - 2y + 3z = -1$$

4. (15 points)

(a) Convert the point $[6, \pi/6, 6]$ from cylindrical to rectangular coordinates.

(b) Convert the point $(3, \pi/6, \pi/3)$ from spherical coordinates to rectangular coordinates.

(c) Convert $\rho = 2\csc\phi\sec\theta$ to rectangular coordinates.

5. (20 points) For $\vec{r}(t) = \langle 1 - 2e^t, 3 + e^t, 4 + 2e^t \rangle$

(a) Find $\vec{r}\,'(t)$

(b) Find the equation of tangent line to $\vec{r}\,(t)$ at $t = 0$

(c) Find the speed at $t = 0$

(d) Find the arc length parametrization. (Use $t = 0$ as the initial point.)

6. (20 points) For $\vec{r}(t) = \langle \frac{t^3}{3} + 1, t^2 + 1, 2t + 5 \rangle$, find

(a) $\vec{v}(t)$ at $t = 1$

(b) $\vec{a}(t)$ at $t = 1$

(c) $\vec{T}(t)$ at $t = 1$ (Hint: $\vec{T}(t) = \frac{\vec{v}(t)}{||\vec{v}(t)||}$)

continued on next page.

(d) $\vec{N}(t)$ at $t = 1$ (Hint: $\vec{N}(t) = \frac{\vec{T}'(t)}{||\vec{T}'(t)||}$)

(e) a_T at $t = 1$ (Hint: $a_T = \frac{\vec{v} \cdot \vec{a}}{||\vec{v}||}$)

(f) a_N at $t = 1$ (Hint: $a_N = \frac{||\vec{v} \times \vec{a}||}{||\vec{v}||}$)

(g) Find the curvature κ at $t = 1$.(Hint: $\kappa = \frac{||\vec{v} \times \vec{a}||}{||\vec{v}||^3}$)

FORMULA PAGE

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin 2x = 2\sin x\cos x$$

$$\cos 2x = \cos^2 x - \sin^2 x$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \sec^2 x$$

$$(\sec x)' = \sec x\tan x$$

$$(\csc x)' = -\csc x\cot x$$

$$(\cot x)' = -\csc^2 x$$

$$(e^x)' = e^x$$

$$(\ln x)' = \frac{1}{x}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arcsec} x)' = \frac{1}{|x|\sqrt{x^2-1}}$$

$$S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2 + \left(\frac{dz}{dt}\right)^2}\, dt$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

$$\sum_{k=1}^n k = \frac{n(n+1)}{2}$$

$$\sum_{k=1}^n k^2 = \frac{n(n+1)(2n+1)}{6}$$

$$\sum_{k=1}^n k^3 = \left[\frac{n(n+1)}{2}\right]^2$$

$$\int \sec x\, dx = \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x\, dx = \frac{1}{2}\left[\sec x\tan x + \ln|\sec x + \tan x|\right] + C$$

$$\int \csc x\, dx = \ln|\csc x - \cot x| + C$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^\infty \frac{x^n}{n!}, \text{ for all } x$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{n=0}^\infty (-1)^n \frac{x^{2n+1}}{(2n+1)!}, \text{ for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \sum_{n=0}^\infty (-1)^n \frac{x^{2n}}{(2n)!}, \text{ for all } x$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots = \sum_{n=0}^\infty x^n, -1 < x < 1$$

$$D = \frac{||\overrightarrow{PQ} \times \overrightarrow{v}||}{||\overrightarrow{v}||}$$

$$D = \frac{|ax_1 + by_1 + cz_1 - d|}{\sqrt{a^2+b^2+c^2}}$$

$$1 + 1 = 2$$