Math 122 Final

SI:

May 1, 2025

EF:

1 - 2	/20
3 - 4	/20
5 - 6	/20
7 - 8	/20
9 - 10	/20
11- 12	/20
13 - 14	/20
15 - 16	/20
17 - 18	/20
19 - 20	/20
Total	/200

Name____________________________

Directions:

1. No books, notes or answers that are more than 6428 digits long. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.

2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.

3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.

4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

1. (10 points)

   (a) Compute $\displaystyle\int \frac{x+1}{\sqrt{2x+x^2}}\,dx$

   (b) Compute $\displaystyle\int x\cos(2x)\,dx$

2. (10 points)

   (a) Compute $\displaystyle\int (\tan^{2025} x)(\sec^4 x)\,dx$

   (b) Compute $\displaystyle\int \frac{1}{(4-x^2)^{3/2}}\,dx$

3. (10 points)

(a) Compute $\displaystyle\int \frac{5x}{(2x+1)(x-2)}\,dx$

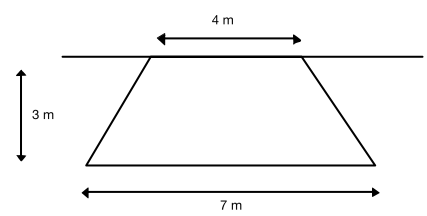
(b) Compute $\displaystyle\int_{-\infty}^{\infty} \frac{e^x}{1+e^{2x}}\,dx$

4. (10 points) For the function $p(x) = C(2x - x^3)$ for $0 \leq x \leq \sqrt{2}$.

(a) Find the value of C so the $p(x)$ is a probability density function.

(b) Find the mean μ.

5. A vertical flat plate in the form of a trapezoid (see below) is submerged in an unknown fluid. If the force on plate is 54,675N, find the density (ρg) in N/m^3 of the fluid.



6. (10 points) Find the length of the curve $f(x) = \ln(\cos x)$ from $[0, \pi/4]$.

7. (10 points) Find the solution of

$$y' + 3x^2\, y = 6x^2 \qquad y(0) = 5$$

8. (10 points) The number of Electric Vehicles (EVs) in the U.S. follows the logistic growth model where, t is measured in years:

$$y' = ky\left(1 - \frac{y}{A}\right)$$

In the year 2020, there were 2 million EVs. By 2025, that number had increased to 8 million. The estimated maximum capacity for EVs is 100 million.

Find:

(a) The initial value y_0

(b) The carrying capacity A.

(c) The solution of the differential equation $y(t)$

(d) The projected number of EVs in 2030?

9. (10 points) A tank contains 20 gallons of pure water. Water containing 2 pounds of Grape Kool-aid mix dissolved per gallon enters the tank at 3 gallons per minute. The well-stirred mixture drains out at 3 gallons per minute.

   (a) Write a differential equation (with initial condition) for how much Grape Kool-aid mix is in the tank at any time.

   (b) Solve the differential equation from part (a).

   (c) How much of the Grape Kool-aid mix will be in the tank after 10 minutes.

10. (10 points)

   (a) Indicate whether the following statements are true or false by circling the appropriate letter. A statement which is sometimes true and sometimes false should be marked false.

a)	If $\lim\limits_{n\to\infty} \lvert a_n \rvert = 0$ then $\lim\limits_{n\to\infty} a_n = 0$	**T**	**F**
b)	If $\lim\limits_{n\to\infty} a_n = 0$ then $\lim\limits_{n\to\infty} a_n^2 = 0$	**T**	**F**
c)	If the sequence a_n is increasing and bounded from above, then it has a limit.	**T**	**F**

   (b) Find the sum of the series: $\displaystyle\sum_{n=0}^{\infty} \frac{5^n + (-2)^n}{7^n}$

11. (10 points) Determine if the following series converge or diverge. For each test you use, you must name the test, perform the test, and state the conclusion you reached from that test.

(a) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3}{(\ln 3)^n}$

(b) $\displaystyle\sum_{n=1}^{\infty} \frac{2n+1}{n^2+3n}$

12. (10 points) Determine if the following series converge absolutely, converge conditionally, or diverge. For each test you use, you must name the test, perform the test, and state the conclusion you reached from that test.

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^n \left(\frac{3^n}{e^n + 3^n}\right)$

13. (10 points) Consider the power series:

$$\sum_{n=1}^{\infty} \frac{(-1)^n (x-2)^n}{3^n \sqrt{n}}$$

(a) Where is the power series centered?

(b) Find the radius of convergence.

(c) Find the interval of convergence. [Hint: Check the endpoints.]

14. (10 points) If the Maclaurin series for $f(x)$ is:

$$f(x) = 1 + 2x + 4x^2 + 8x^3 \ldots = \sum_{n=0}^{\infty} (2x)^n$$

(a) What is $f(0)$?

(b) What is $f(\frac{1}{3})$?

(c) Is $f(x)$ increasing or decreasing at $x = 0$? Why?

(d) $f^{2025}(0)$ the 2025th derivative of $f(x)$ at $x = 0$?

15. (10 points) For the curve given by:

$$c(t) = \left(t^2, t^3 - 3t\right)$$

Find

(a) $\frac{dy}{dx}$

(b) the equation(s) of a tangent line(s) at the point $(3, 0)$ (Hint: there may be more that one)

16. (10 points) For the the area of region inside $r = 4\sin\theta$ but outside $r = 2$

Fill in the boxes.

$$A = \int_{\boxed{}}^{\boxed{}} \frac{\left(\boxed{}\right)^2}{2}\, d\theta \quad - \quad \int_{\boxed{}}^{\boxed{}} \frac{\left(\boxed{}\right)^2}{2}\, d\theta$$

17. (10 points)

(a) Find an equation of the sphere with radius 2 and center $(1, -1, 3)$

(b) If $P(3, 3, 4)$ and $Q(1, 5, 5)$, find the distance from P to Q.

(c) If $P(3, 3, 4)$ and $Q(1, 5, 5)$, find a unit vector in the direction of $\vec{PQ}$

18. (10 points) Indicate whether the following statements are true or false by circling the appropriate letter. A statement which is sometimes true and sometimes false should be marked false.

a)	The dot product of two unit vectors is one.	**T**	**F**
b)	For vectors $\overrightarrow{A}$ and $\overrightarrow{B}$ then $\overrightarrow{A} \times \overrightarrow{B} = \overrightarrow{B} \times \overrightarrow{A}$	**T**	**F**
c)	If $\overrightarrow{A} \cdot \overrightarrow{B} = 0$ then either $\overrightarrow{A} = \overrightarrow{0}$ or $\overrightarrow{B} = \overrightarrow{0}$	**T**	**F**
d)	If $\overrightarrow{A} \times \overrightarrow{B} = \vec{0}$ then either $\overrightarrow{A} = \overrightarrow{0}$ or $\overrightarrow{B} = \overrightarrow{0}$	**T**	**F**
e)	If $\overrightarrow{A} \times \overrightarrow{B} = \overrightarrow{A} \times \overrightarrow{C}$ then $\overrightarrow{B} = \overrightarrow{C}$	**T**	**F**

19. (10 points)

(a) Find the parametric equation of the line through the points $(1, 1, 1)$ and $(1, 2, 3)$

(b) Are the lines

$$x = 1 + t \qquad y = 3 - t \qquad z = 1 + t$$

and

$$x = -1 + t \qquad y = 5 - t \qquad z = 8 - 2t$$

parallel, intersect (find the point of intersection), or skew?

20. (10 points)

(a) Where does the line with parametric equations:

$$x = -1 + 3t \qquad y = 2 - 2t \qquad z = 3 + t$$

intersect the plane

$$3x + y - 4z = -4$$

(b) Find the distance from the point $(8, 2, 3)$ to the plane

$$x - 2y + 2z = 1$$

FORMULA PAGE

$$\sin^2\theta+\cos^2\theta=1$$
$$\tan^2\theta+1=\sec^2\theta$$
$$1+\cot^2\theta=\csc^2\theta$$
$$\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$$
$$\sin(\alpha-\beta)=\sin\alpha\cos\beta-\cos\alpha\sin\beta$$
$$\cos(\alpha+\beta)=\cos\alpha\cos\beta-\sin\alpha\sin\beta$$
$$\cos(\alpha-\beta)=\cos\alpha\cos\beta+\sin\alpha\sin\beta$$
$$\tan(\alpha+\beta)=\frac{\tan\alpha+\tan\beta}{1-\tan\alpha\tan\beta}$$
$$\sin^2 x=\frac{1-\cos 2x}{2}$$
$$\cos^2 x=\frac{1+\cos 2x}{2}$$
$$\sin 2x=2\sin x\cos x$$
$$\cos 2x=\cos^2 x-\sin^2 x$$
$$(\sin x)'=\cos x$$
$$(\cos x)'=-\sin x$$
$$(\tan x)'=\sec^2 x$$
$$(\sec x)'=\sec x\tan x$$
$$(\csc x)'=-\csc x\cot x$$
$$(\cot x)'=-\csc^2 x$$
$$(e^x)'=e^x$$
$$(\ln x)'=\frac{1}{x}$$
$$(\arcsin x)'=\frac{1}{\sqrt{1-x^2}}$$
$$(\arctan x)'=\frac{1}{1+x^2}$$
$$(\operatorname{arcsec} x)'=\frac{1}{|x|\sqrt{x^2-1}}$$
$$S=\int_a^b\sqrt{1+(f'(x))^2}\,dx$$
$$SA=\int_a^b 2\pi f(x)\sqrt{1+(f'(x))^2}\,dx$$
$$S=\int_a^b\sqrt{\left(\frac{dx}{dt}\right)^2+\left(\frac{dy}{dt}\right)^2}\,dt$$

$$S=\int_\alpha^\beta\sqrt{r^2+(r')^2}\,d\theta$$
$$F=\int_a^b\rho\ h(x)\ w(x)\ dx$$
$$\cosh^2 x-\sinh^2 x=1$$
$$(\sinh x)'=\cosh x$$
$$(\cosh x)'=\sinh x$$
$$\sinh x=\frac{e^x-e^{-x}}{2}$$
$$\cosh x=\frac{e^x+e^{-x}}{2}$$
$$x_{n+1}=x_n-\frac{f(x_n)}{f'(x_n)}$$
$$f'(c)=\frac{f(b)-f(a)}{b-a}$$
$$f(c)=\frac{1}{b-a}\int_a^b f(x)dx$$
$$\int\sec x\,dx=\ln|\sec x+\tan x|+C$$
$$\int\sec^3 x\,dx=\frac{1}{2}\left[\sec x\tan x+\ln|\sec x+\tan x|\right]+C$$
$$\int\csc x\,dx=\ln|\csc x-\cot x|+C$$
$$e^x=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots=\sum_{n=0}^{\infty}\frac{x^n}{n!},\text{ for all }x$$
$$\sin x=x-\frac{x^3}{3!}+\frac{x^5}{5!}+\cdots=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n+1}}{(2n+1)!},\text{ for all }x$$
$$\cos x=1-\frac{x^2}{2!}+\frac{x^4}{4!}+\cdots=\sum_{n=0}^{\infty}(-1)^n\frac{x^{2n}}{(2n)!},\text{ for all }x$$
$$\frac{1}{1-x}=1+x+x^2+x^3+x^4+\cdots=\sum_{n=0}^{\infty}x^n,-1<x<1$$
$$y=\frac{A}{1-\dfrac{e^{-kt}}{B}}\qquad B=\frac{y_0}{y_0-A}$$
$$D=\frac{|ax_1+by_1+cz_1-d|}{\sqrt{a^2+b^2+c^2}}$$
$$1+1=2$$

NAME	STATEMENT	COMMENTS
Geometric Series	$$\sum_{n=0}^{\infty} ar^n$$ Converges if $\lvert r\rvert < 1$ and has sum $S = \dfrac{a}{1-r}$ Diverges if $\lvert r\rvert \geq 1$	
p-series	$$\sum_{n=1}^{\infty} \frac{1}{n^p}$$ Converges if $p > 1$, diverges if $p \leq 1$	
n-th Term Test or Divergence Test	If $\lim\limits_{n\to\infty} a_n \neq 0$, the $\sum\limits_{n=1}^{\infty} a_n$ diverges.	If $\lim\limits_{n\to\infty} a_n = 0$, $\sum\limits_{n=1}^{\infty} a_n$ may or may not converge.
Integral Test	Let $\sum\limits_{n=1}^{\infty} a_n$ be a series with positive terms, and let $f(x)$ be the function that results when n is replaced by x in the formula for a_n. If f is decreasing and continuous for $x \geq 1$, then $\sum\limits_{n=1}^{\infty} a_n$ and $\int\limits_{1}^{\infty} f(x)\, dx$ both converge or both diverge.	Use this test when $f(x)$ is easy to integrate.
Comparison Test	Let $\sum\limits_{n=1}^{\infty} a_n$ and $\sum\limits_{n=1}^{\infty} b_n$ be a series with positive terms such that if $a_n < b_n$ and $\sum\limits_{n=1}^{\infty} b_n$ converges then $\sum\limits_{n=1}^{\infty} a_n$ converges or if $b_n < a_n$ and $\sum\limits_{n=1}^{\infty} b_n$ diverges then $\sum\limits_{n=1}^{\infty} a_n$ diverges	

NAME	STATEMENT	COMMENTS
Limit Comparison Test	Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be a series with positive terms such that $$\lim_{n\to\infty} \frac{a_n}{b_n} = \rho$$ If $0 < \rho < \infty$, then both series converge or both diverge.	
Ratio Test	Let $\sum_{n=1}^{\infty} a_n$ be a series with positive terms and suppose $$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \rho$$ a) Series converges if $\rho < 1$. b) Series diverges if $\rho > 1$. c) No conclusion if $\rho = 1$.	Try this test when a_n involves factorials or n-th powers.
Root Test	Let $\sum_{n=1}^{\infty} a_n$ be a series with positive terms and suppose $$\lim_{n\to\infty} \sqrt[n]{a_n} = \rho$$ a) Series converges if $\rho < 1$. b) Series diverges if $\rho > 1$. c) No conclusion if $\rho = 1$.	Try this test when a_n involves nth powers
Alternating Series Test	Let $\sum_{n=1}^{\infty} a_n$ be a series with alternating terms if $$\lim_{n\to\infty} a_n = 0$$ and $$\lvert a_n\rvert > \lvert a_{n+1}\rvert$$ the series converges conditionally	This test applies only to alternating series.