Math 122 Test 3

November 18, 2025

SI: EF:

1	
2	
3	
4	
5	
Total	

Name______________________________

Directions:

1. No books, notes or playing Christmas songs before Thanksgiving. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.

2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.

3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.

4. Numerical experiments do not count as justification. For example, computing the first few terms of a series is not enough to show the terms decrease. Computing the first few partial sums of a series is not enough to show the series converges or diverges. Plugging in numbers is not enough to justify the computation of a limit.

5. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

1. (20 points)

(a) Find the limit of the following sequence:

$$\left\{ \frac{6n^2}{3n+2} - \frac{2n^2}{n+2} \right\}_{n=1}^{\infty}$$

(b) Find the sum of the following series:

$$5 - \frac{5}{3} + \frac{5}{9} - \frac{5}{27} + \frac{5}{81} - \frac{5}{243} \cdots$$

(c) Determine if the following series converges or diverges, and if it converges, find the sum.

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n^2+n}}$$

2. (20 points)

(a) For each of the following series, determine if it converges or diverges. For each test you use, you must name the test, perform the test, and state the conclusion you reached from that test.

i. $\displaystyle\sum_{n=2}^{\infty} \frac{1}{\sqrt{n} - \sqrt[3]{n}}$

ii. $\displaystyle\sum_{n=1}^{\infty} \frac{n^2}{e^n}$

(b) Determine if the following series converge absolutely, converge conditionally, or diverge. For each test you use, you must name the test, perform the test, and state the conclusion you reached from that test.

i. $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+5}$

ii. $\displaystyle\sum_{n=10}^{\infty} (-1)^n \frac{1}{n^{\ln n}}$

3. (20 points)

(a) Consider the power series:

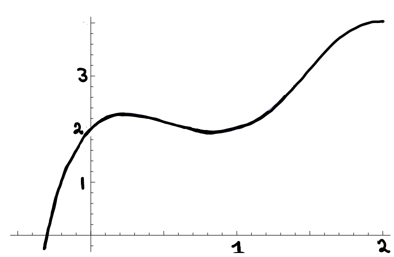
$$\sum_{n=1}^{\infty} \frac{(x-4)^n}{(-3)^n \sqrt{n}}$$

i. Where is the power series centered?

ii. Find the radius of convergence.

iii. Find the interval of convergence. [Hint: Check the endpoints.]

(b) Below is the graph of $y = f(x)$



Which of the following could be the second degree Taylor polynomial for $f(x)$ at $a = 1$.

i. $T_2(x) = 2 - (x-1) + 3(x-1)^2$

ii. $T_2(x) = 2 - (x-1) - 3(x-1)^2$

iii. $T_2(x) = 2 + (x-1) + 3(x-1)^2$

iv. $T_2(x) = 2 + (x-1) - 3(x-1)^2$

4. (20 points) For the curve

$$x = \frac{\sqrt{3}}{2}t^2 \qquad y = t - \frac{t^3}{4}$$

(a) Find $\frac{dy}{dx}$.

(b) Find the equations of the tangent lines at the point $\left(\frac{\sqrt{3}}{2}, \frac{3}{4}\right)$.

(c) The speed at $t = 1$.

(d) The arc length for $0 \leq t \leq 2$.

5. (20 points)

(a) i. Find the Maclaurin Series for $f(x) = \dfrac{x}{(1-x)^2}$

Hint: $\dfrac{d}{dx}\left(\dfrac{1}{1-x}\right) = \dfrac{1}{(1-x)^2}$

ii. Find $\displaystyle\sum_{n=1}^{\infty} \frac{n}{2^n}$. Hint: Let $x = \frac{1}{2}$

(b) For the area inside $r = 2 + 2\sin\theta$ and outside $r = 3$

Fill in the boxes:

$$A = \int_{\boxed{}}^{\boxed{}} \frac{\left(\boxed{}\right)^2}{2}\, d\theta \quad - \quad \int_{\boxed{}}^{\boxed{}} \frac{\left(\boxed{}\right)^2}{2}\, d\theta$$

FORMULA PAGE

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\sin(\alpha-\beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha+\beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\cos(\alpha-\beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin^2 x = \frac{1-\cos 2x}{2}$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$\sin 2x = 2\sin x\cos x$$

$$(\sin x)' = \cos x$$

$$(\cos x)' = -\sin x$$

$$(\tan x)' = \sec^2 x$$

$$(\sec x)' = \sec x\tan x$$

$$(\csc x)' = -\csc x\cot x$$

$$(\cot x)' = -\csc^2 x$$

$$(e^x)' = e^x$$

$$(\ln x)' = \frac{1}{x}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1-x^2}}$$

$$(\arctan x)' = \frac{1}{1+x^2}$$

$$(\operatorname{arcsec} x)' = \frac{1}{|x|\sqrt{x^2-1}}$$

$$(\sinh x)' = \cosh x$$

$$(\cosh x)' = \sinh x$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$

$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$

$$f(c) = \frac{1}{b-a}\int_a^b f(x)dx$$

$$\int \sec x\,dx = \ln|\sec x + \tan x| + C$$

$$\int \sec^3 x\,dx = \frac{1}{2}\left[\sec x\tan x + \ln|\sec x + \tan x|\right] + C$$

$$\int \csc x\,dx = \ln|\csc x - \cot x| + C$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{k=0}^{\infty}\frac{x^k}{k!}, \text{ for all } x$$

$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{k=0}^{\infty}(-1)^k\frac{x^{2k+1}}{(2k+1)!}, \text{ for all } x$$

$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \sum_{k=0}^{\infty}(-1)^k\frac{x^{2k}}{(2k)!}, \text{ for all } x$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + x^4 + \cdots = \sum_{k=0}^{\infty} x^k, -1 < x < 1$$

$$1 + 1 = 2$$

NAME	STATEMENT	COMMENTS
Geometric Series	$$\sum_{k=0}^{\infty} ar^k$$ Converges if $\lvert r\rvert < 1$ and has sum $S = \frac{a}{1-r}$ Diverges if $\lvert r\rvert \geq 1$	
p-series	$$\sum_{k=1}^{\infty} \frac{1}{k^p}$$ Converges if $p > 1$, diverges if $p \leq 1$	
Divergence Test	If $\lim\limits_{k\to\infty} a_k \neq 0$, the $\sum\limits_{k=1}^{\infty} a_k$ diverges.	If $\lim\limits_{n\to\infty} a_k = 0$, $\sum\limits_{k=1}^{\infty} a_k$ may or may not converge.
Integral Test	Let $\sum\limits_{k=1}^{\infty} a_k$ be a series with positive terms, and let $f(x)$ be the function that results when k is replaced by x in the formula for a_k. If f is decreasing and continuous for $x \geq 1$, then $\sum\limits_{k=1}^{\infty} a_k$ and $\int\limits_1^{\infty} f(x)\, dx$ both converge or both diverge.	Use this test when $f(x)$ is easy to integrate.
Comparison Test	Let $\sum\limits_{k=1}^{\infty} a_k$ and $\sum\limits_{k=1}^{\infty} b_k$ be a series with positive terms such that if $a_k < b_k$ and $\sum\limits_{k=1}^{\infty} b_k$ converges then $\sum\limits_{k=1}^{\infty} a_k$ converges or if $b_k < a_k$ and $\sum\limits_{k=1}^{\infty} b_k$ diverges then $\sum\limits_{k=1}^{\infty} a_k$ diverges	

NAME	STATEMENT	COMMENTS
Limit Comparison Test	Let $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ be a series with positive terms such that $$\lim_{k\to\infty} \frac{a_k}{b_k} = \rho$$ If $0 < \rho < \infty$, then both series converge or both diverge.	
Ratio Test	Let $\sum_{k=1}^{\infty} a_k$ be a series with positive terms and suppose $$\lim_{k\to\infty} \frac{a_{k+1}}{a_k} = \rho$$ a) Series converges if $\rho < 1$. b) Series diverges if $\rho > 1$. c) No conclusion if $\rho = 1$.	Try this test when a_k involves factorials or k-th powers.
Root Test	Let $\sum_{k=1}^{\infty} a_k$ be a series with positive terms and suppose $$\lim_{k\to\infty} \sqrt[k]{a_k} = \rho$$ a) Series converges if $\rho < 1$. b) Series diverges if $\rho > 1$. c) No conclusion if $\rho = 1$.	Try this test when a_k involves kth powers
Alternating Series Test	Let $\sum_{k=1}^{\infty} a_k$ be a series with alternating terms if $$\lim_{k\to\infty} a_k = 0$$ and $$\lvert a_k\rvert > \lvert a_{k+1}\rvert$$ the series converges conditionally	This test applies only to alternating series.