

SI:

Math 122 Test 3

November 18, 2025

EF:

1	
2	
3	
4	
5	
Total	

Name KEY

Directions:

1. No books, notes or playing Christmas songs before Thanksgiving. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.
2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.
3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.
4. Numerical experiments do not count as justification. For example, computing the first few terms of a series is not enough to show the terms decrease. Computing the first few partial sums of a series is not enough to show the series converges or diverges. Plugging in numbers is not enough to justify the computation of a limit.
5. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

1. (20 points)

(a) Find the limit of the following sequence:

⑥

$$\left\{ \frac{6n^2}{3n+2} - \frac{2n^2}{n+2} \right\}_{n=1}^{\infty}$$
$$= \lim_{N \rightarrow \infty} \frac{6N^2(N+2) - 2N^2(3N+2)}{(3N+2)(N+2)}$$
$$= \lim_{N \rightarrow \infty} \frac{\cancel{6N^3} + 12N^2 - \cancel{6N^3} - 4N^2}{(3N+2)(N+2)} = \frac{8}{3}$$

⑦

(b) Find the sum of the following series:

$$5 - \frac{5}{3} + \frac{5}{9} - \frac{5}{27} + \frac{5}{81} - \frac{5}{243} \dots$$
$$= \sum_{N=0}^{\infty} 5 \left(-\frac{1}{3} \right)^N = \frac{5}{1 - -\frac{1}{3}} = \frac{5}{\frac{4}{3}} = \frac{15}{4}$$

⑦

(c) Determine if the following series converges or diverges, and if it converges, find the sum.

$$\sum_{n=1}^{\infty} \frac{\sqrt{n+1} - \sqrt{n}}{\sqrt{n^2+n}}$$
$$= \sum_{N=1}^{\infty} \left(\frac{\cancel{\sqrt{N+1}}}{\sqrt{N} \cancel{\sqrt{N+1}}} - \frac{\cancel{\sqrt{N}}}{\cancel{\sqrt{N}} \sqrt{N+1}} \right) = \sum_{N=1}^{\infty} \left(\frac{1}{\sqrt{N}} - \frac{1}{\sqrt{N+1}} \right)$$
$$= \left(1 - \cancel{\frac{1}{\sqrt{2}}} \right) + \left(\cancel{\frac{1}{\sqrt{2}}} - \cancel{\frac{1}{\sqrt{3}}} \right) + \left(\cancel{\frac{1}{\sqrt{3}}} - \cancel{\frac{1}{\sqrt{4}}} \right) + \dots$$
$$= 1 \quad \text{CONVERGES}$$

2. (20 points)

- (a) For each of the following series, determine if it converges or diverges. For each test you use, you must name the test, perform the test, and state the conclusion you reached from that test.

⑤ i. $\sum_{n=2}^{\infty} \frac{1}{\sqrt{n} - \sqrt[3]{n}}$ Comp TEST $\frac{1}{\sqrt{N} - \sqrt[3]{N}} > \frac{1}{\sqrt{N}}$

$\lesssim \frac{1}{\sqrt{N}}$ DIV. $p = \frac{1}{2} < 1$ DIVERGES

⑤ ii. $\sum_{n=1}^{\infty} \frac{n^2}{e^n}$ RATIO TEST

$\lim_{N \rightarrow \infty} \frac{(N+1)^2}{e^{N+1}} = \frac{1}{e} < 1$ CONVERGES

- (b) Determine if the following series converge absolutely, converge conditionally, or diverge. For each test you use, you must name the test, perform the test, and state the conclusion you reached from that test.

⑤ i. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2 + 5}$ L.C.T. $\lesssim \frac{1}{N}$

$\lim_{N \rightarrow \infty} \frac{\frac{N}{N^2+1}}{\frac{1}{N}} = \lim_{N \rightarrow \infty} \frac{N^2}{N^2+1} = 1$ GOTO AST

$\lim_{N \rightarrow \infty} \frac{N}{N^2+1} = 0 \checkmark$ $\frac{N+1}{(N+1)^2+5} < \frac{N}{N^2+5} \checkmark$

⑤ ii. $\sum_{n=10}^{\infty} (-1)^n \frac{1}{n^{\ln n}}$ CONV. COND

Comp TEST

$\frac{1}{N^{\ln N}} < \frac{1}{N^2} \lesssim \frac{1}{N^2}$ CONV.

CONV. ABS

3. (20 points)

(a) Consider the power series:

$$\sum_{n=1}^{\infty} \frac{(x-4)^n}{(-3)^n \sqrt{n}}$$

i. Where is the power series centered?

②

$$C = 4$$

ii. Find the radius of convergence.

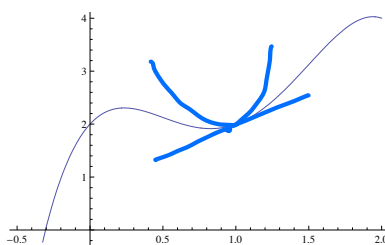
⑥ $\lim_{N \rightarrow \infty} \left| \frac{(x-4)^{N+1} \sqrt{N}}{(-3)^{N+1} \sqrt{N+1} (x-4)^N} \right| = \left| \frac{x-4}{-3} \right| < 1 \quad |x-4| < 3$
 $R = 3$

iii. Find the interval of convergence. [Hint: Check the endpoints.]

⑥ $\left(\begin{array}{l} x=1 \quad \sum_{n=1}^{\infty} \frac{(1-4)^n}{(-3)^n \sqrt{n}} = \sum \frac{1}{\sqrt{n}} \text{ DIV.} \\ x=7 \quad \sum_{n=1}^{\infty} \frac{(7-4)^n}{(-3)^n \sqrt{n}} = \sum \frac{(-1)^n}{\sqrt{n}} \text{ CONV. COND} \end{array} \right. \quad (1, 7]$

(b) Below is the graph of $y = f(x)$

⑥



② $f(1) = 2$

② $f'(1) > 0$

② $f''(1) > 0$

Which of the following could be the second degree Taylor polynomial for $f(x)$ at $a = 1$.

i. $T_2(x) = 2 - (x-1) + 3(x-1)^2$

ii. $T_2(x) = 2 - (x-1) - 3(x-1)^2$

iii. $T_2(x) = 2 + (x-1) + 3(x-1)^2$

iv. $T_2(x) = 2 + (x-1) - 3(x-1)^2$

4. (20 points) For the curve

$$x = \frac{\sqrt{3}}{2}t^2 \quad y = t - \frac{t^3}{4}$$

(a) Find $\frac{dy}{dx}$.

⑤
$$\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{1 - \frac{3t^2}{4}}{\sqrt{3}t}$$

(b) Find the equations of the tangent lines at the point $\left(\frac{\sqrt{3}}{2}, \frac{3}{4}\right)$.

⑤
$$t=1 \quad \frac{dy}{dx} = \frac{1 - \frac{3}{4}}{\sqrt{3}} = \frac{\frac{1}{4}}{\sqrt{3}} = \frac{\sqrt{3}}{12}$$

$$y - \frac{3}{4} = \frac{\sqrt{3}}{12} \left(x - \frac{\sqrt{3}}{2} \right)$$

(c) The speed at $t = 1$.

⑤
$$\begin{aligned} \frac{ds}{dt} &= \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{(\sqrt{3}t)^2 + \left(1 - \frac{3t^2}{4}\right)^2} \\ &= \sqrt{3t^2 + 1 - \frac{3t^2}{2} + \frac{9t^4}{16}} = \sqrt{\left(1 + \frac{3t^2}{4}\right)^2} = 1 + \frac{3t^2}{4} \Big|_{t=1} \\ &= \frac{7}{4} \end{aligned}$$

⑤ (d) The arc length for $0 \leq t \leq 2$.

$$\begin{aligned} S &= \int_0^2 \left(1 + \frac{3}{4}t^2\right) dt = t + \frac{t^3}{4} \Big|_0^2 \\ &= 2 + 2 = 4 \end{aligned}$$

5. (20 points)

(5) (a) i. Find the Maclaurin Series for $f(x) = \frac{x}{(1-x)^2}$

Hint: $\frac{d}{dx} \left(\frac{1}{1-x} \right) = \frac{1}{(1-x)^2}$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \dots$$

$$\frac{1}{(1-x)^2} = 1 + 2x + 3x^2 + 4x^3 + \dots$$

$$\frac{x}{(1-x)^2} = x + 2x^2 + 3x^3 + 4x^4 + \dots$$

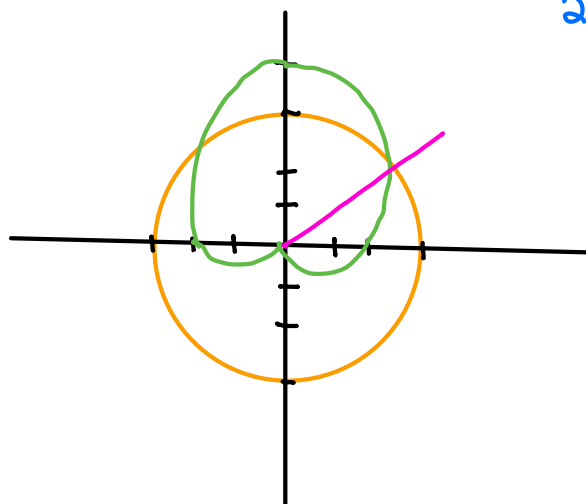
(5) ii. Find $\sum_{n=1}^{\infty} \frac{n}{2^n}$. Hint: Let $x = \frac{1}{2}$

$$\sum_{n=1}^{\infty} \frac{n}{2^n} = \left. \frac{x}{(1-x)^2} \right|_{x=\frac{1}{2}} = \frac{\frac{1}{2}}{(1-\frac{1}{2})^2} = \frac{\frac{1}{2}}{\frac{1}{4}} = 2$$

(b) For the area inside $r = 2 + 2 \sin \theta$ and outside $r = 3$

Fill in the boxes:

$$A = \int_{\boxed{\pi/6}}^{\boxed{5\pi/6}} \frac{(\boxed{2+2\sin\theta})^2}{2} d\theta - \int_{\boxed{\pi/6}}^{\boxed{5\pi/6}} \frac{(\boxed{3})^2}{2} d\theta$$



$$\begin{aligned} 2 + 2\sin\theta &= 3 \\ \sin\theta &= \frac{1}{2} \\ \theta &= \frac{\pi}{6}, \frac{5\pi}{6} \end{aligned}$$