Directions:

1. No book, notes, or winter weather advisories. You may use a calculator to do routine arithmetic computations. You may not use your calculator to store notes or formulas. You may not share a calculator with anyone.

2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.

3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.

4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

Have a Safe and Happy Break
1. (10 points)

   (a) Compute \( \int \frac{\ln x}{x^3} \, dx \)

   (b) Compute \( \int_0^{\pi/4} (\sec^4 x)(\tan x) \, dx \)

2. (10 points)

   (a) Compute \( \int \frac{1}{x^2\sqrt{x^2 - 4}} \, dx \)

   (b) Compute \( \int \sinh^3 x \, dx \)
3. (10 points)

(a) Compute \( \int \frac{x + 4}{x^2 - 5x + 6} \, dx \)

(b) Compute \( \int_{-1}^{4} \frac{1}{(x - 1)^4} \, dx \)

4. (10 points) Find the length of the curve \( y = \frac{1}{3} x^{3/2} - x^{1/2} \) for \( 1 \leq x \leq 9 \).
5. An isosceles triangle with base 1 meter and height 2 meters is submerged vertically in water ($\rho g = 9810 \text{ N/m}^3$) with the vertex at the surface. Find the force on the plate.

6. (10 points) Find the centroid of the region lying between $y = x^2$ and $y = 4$. 
7. (10 points) For the differential equation \( \frac{dy}{dx} = F(x, y) \), where \( F(x, y) \) is given below:

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(a) Use Euler’s method with \( h = 1 \) and \( y(0) = 2 \) to approximate \( y(4) \).

(b) Sketch the slope field at the given points.

8. (10 points)

(a) Solve the initial value problem

\[ xy' + 5y = 7x^2 \quad y(2) = 5 \]

(b) Find the general solution of

\[ y' = 2x \sqrt{y - 1} \]
9. (10 points) Bailey wants to change the oil in her engine. Her engine has 300 quarts of oil. To change the oil while the engine is running, a machine is attached that removes 15 quarts of oil each minute from the engine and replaces it with 15 quarts of clean oil. Let $y = y(t)$ denote the amount of clean oil in the engine at time $t$, with $y(0) = 0$.

(a) Write a differential equation for how much new oil is in circulation.

(b) Solve the differential equation from part (a).

(c) How long will it take for 200 quarts of the oil in circulation to be clean oil?

10. (10 points)

(a) Determine the limit of the sequence $\left\{ \frac{1 + (-1)^n}{n} \right\}_{n=1}^{\infty}$

(b) Find the sum of $\sum_{n=1}^{\infty} \left[ \cos \left( \frac{1}{n} \right) - \cos \left( \frac{1}{n + 1} \right) \right]$
11. (10 points) For each of the following series, determine if it converges or diverges. For each test you use, you must name the test, perform the test, and state the conclusion you reached from that test.

(a) \[ \sum_{n=1}^{\infty} \frac{e^n}{2^{2n}} \]

(b) \[ \sum_{n=1}^{\infty} \frac{n^3 + 2n - 1}{n^5 - n + 10} \]

12. (10 points) For each of the following series, determine if the following series converge absolutely, converge conditionally, or diverge. For each test you use, you must name the test, perform the test, and state the conclusion you reached from that test.

(a) \[ \sum_{n=0}^{\infty} \frac{\cos(\pi n)}{\sqrt{n + 1}} \]

(b) \[ \sum_{n=2}^{\infty} \frac{(-1)^n}{n(ln n)^2} \]
13. (10 points) Find the radius and interval of convergence of the power series

\[ \sum_{n=1}^{\infty} \frac{5^n(2x - 1)^n}{n!} \]

14. (10 points) Consider the function

\[ f(x) = \frac{3x - \sin(3x)}{x^3} \]

(a) Find the first four nonzero terms of the Maclaurin Series of \( f(x) \)

(b) What is \( f^5(0) \) (the fifth derivative of \( f(x) \) at \( x = 0 \))?
15. (10 points) Consider the parametric curve defined by

\[ x = t^2 \quad y = t^3 - t \]

(a) For which values of \( t \) does the curve have a horizontal tangent line?

(b) For which values of \( t \) does the curve have a vertical tangent line?

(c) Find the tangent line at \( t = 2 \).

16. (10 points)

For the area of region inside \( r = 6 \sin \theta \) and outside \( r = 2 + 2 \sin \theta \).

Fill in the boxes.

\[
A = \int_{0}^{2} \left[ \left( \frac{6 \sin \theta}{2} \right)^2 - \left( \frac{2 + 2 \sin \theta}{2} \right)^2 \right] d\theta
\]
17. (10 points) For vectors $\vec{a} = \langle 3, 2, -1 \rangle$ and $\vec{b} = \langle 6, 6, 6 \rangle$ find:

(a) $\vec{a} \cdot \vec{b}$

(b) $\vec{a} \times \vec{b}$

(c) Find a vector $\vec{c} \neq \vec{0}$, where $\vec{a} \cdot \vec{c} = \vec{0}$

(d) Find the area of the triangle with $\vec{a}$ and $\vec{b}$ as two of the sides.

18. Determine if the two lines:

$L_1 : x = 4t - 1, \quad y = t + 3, \quad z = t + 5$

$L_2 : x = -13 + 12t, \quad y = 1 + 6t, \quad z = 2 + 3t$

are parallel, intersect or are skew.
19. (10 points) Find the equation of the plane containing \((1, 2 - 1)\) that is perpendicular to the line of intersection of the planes \(2x + y + z = 2\) and \(x + 2y + z = 3\).

20. (10 points) Indicate whether the following statements are true or false by circling the appropriate letter. A statement which is sometimes true and sometimes false should be marked false.

a) If a sequence is bounded, then it converges. T F

b) The line \(x = t, y = 2t, z = 3t\) and the plane \(x + y - z = 5\) are perpendicular. T F

c) If \(\vec{a} \cdot \vec{a} = \vec{b} \cdot \vec{b}\), then \(||\vec{a}|| = ||\vec{b}||\) T F

d) If \(\vec{a} \times \vec{a} = \vec{b} \times \vec{b}\), then \(\vec{a} = \vec{b}\) T F

e) \(1 + 1 = 2\) T F
\[
\sin^2 \theta + \cos^2 \theta = 1
\]
\[
\tan^2 \theta + 1 = \sec^2 \theta
\]
\[
1 + \cot^2 \theta = \csc^2 \theta
\]
\[
\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta
\]
\[
\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta
\]
\[
\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta
\]
\[
\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta
\]
\[
\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}
\]
\[
\sin^2 x = \frac{1 - \cos 2x}{2}
\]
\[
\cos^2 x = \frac{1 + \cos 2x}{2}
\]
\[
\sin 2x = 2 \sin x \cos x
\]
\[
\cos 2x = \cos^2 x - \sin^2 x
\]
\[
(\sin x)' = \cos x
\]
\[
(\cos x)' = -\sin x
\]
\[
(\tan x)' = \sec^2 x
\]
\[
(\sec x)' = \tan x \sec x
\]
\[
(\csc x)' = -\csc x \cot x
\]
\[
(\cot x)' = -\csc^2 x
\]
\[
(e^x)' = e^x
\]
\[
(\ln x)' = \frac{1}{x}
\]
\[
(\arcsin x)' = \frac{1}{\sqrt{1 - x^2}}
\]
\[
(\arctan x)' = \frac{1}{1 + x^2}
\]
\[
(\arccsc x)' = \frac{1}{|x| \sqrt{x^2 - 1}}
\]
\[
S = \int_a^b \sqrt{1 + (f'(x))^2} \, dx
\]
\[
S = \int_a^b \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} \, dt
\]
\[
S = \int_a^b \sqrt{r'^2 + (r^2)^2} \, d\theta
\]
\[
F = \int_a^b \rho \, h(x) \, w(x) \, dx
\]
\[
(\sin x)' = \cosh x
\]
\[
(\cosh x)' = \sinh x
\]
\[
\sinh x = \frac{e^x - e^{-x}}{2}
\]
\[
\cosh x = \frac{e^x + e^{-x}}{2}
\]
\[
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}
\]
\[
f'(c) = \frac{f(b) - f(a)}{b - a}
\]
\[
\int \sec x \, dx = \ln |\sec x + \tan x| + C
\]
\[
\int \sec^3 x \, dx = \frac{1}{2} [\sec x \tan x + \ln |\sec x + \tan x|] + C
\]
\[
\int \csc x \, dx = \ln |\csc x - \cot x| + C
\]
\[
e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for all } x
\]
\[
\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n+1}}{(2n+1)!}, \text{ for all } x
\]
\[
\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots = \sum_{n=0}^{\infty} (-1)^n \frac{x^{2n}}{(2n)!}, \text{ for all } x
\]
\[
\frac{1}{1 - x} = 1 + x + x^2 + x^3 + x^4 + \cdots = \sum_{n=0}^{\infty} x^n, -1 < x < 1
\]
\[
y = \frac{A}{1 - e^{-kt}} \quad B = \frac{y_0}{y_0 - A}
\]
\[
D = \frac{|ax_1 + by_1 + cz_1 - d|}{\sqrt{a^2 + b^2 + c^2}}
\]
\[
1 + 1 = 2
\]
<table>
<thead>
<tr>
<th>NAME</th>
<th>STATEMENT</th>
<th>COMMENTS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Geometric Series</td>
<td>$\sum_{n=0}^{\infty} ar^n$</td>
<td>Converges if $</td>
</tr>
<tr>
<td>$p$-series</td>
<td>$\sum_{n=1}^{\infty} \frac{1}{n^p}$</td>
<td>Converges if $p &gt; 1$, diverges if $p \leq 1$</td>
</tr>
<tr>
<td>$n$-th Term Test or Divergence Test</td>
<td>If $\lim_{n \to \infty} a_n \neq 0$, the $\sum_{n=1}^{\infty} a_n$ diverges.</td>
<td>If $\lim_{n \to \infty} a_n = 0$, $\sum_{n=1}^{\infty} a_n$ may or may not converge.</td>
</tr>
<tr>
<td>Integral Test</td>
<td>Let $\sum_{n=1}^{\infty} a_n$ be a series with positive terms, and let $f(x)$ be the function that results when $n$ is replaced by $x$ in the formula for $a_n$. If $f$ is decreasing and continuous for $x \geq 1$, then $\sum_{n=1}^{\infty} a_n$ and $\int_{1}^{\infty} f(x) , dx$ both converge or both diverge.</td>
<td>Use this test when $f(x)$ is easy to integrate.</td>
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<tr>
<td>Comparison Test</td>
<td>Let $\sum_{n=1}^{\infty} a_n$ and $\sum_{n=1}^{\infty} b_n$ be a series with positive terms such that if $a_n &lt; b_n$ and $\sum_{n=1}^{\infty} b_n$ converges then $\sum_{n=1}^{\infty} a_n$ converges or if $b_n &lt; a_n$ and $\sum_{n=1}^{\infty} b_n$ diverges then $\sum_{n=1}^{\infty} a_n$ diverges</td>
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<td>Limit Comparison Test</td>
<td>Let ( \sum_{n=1}^{\infty} a_n ) and ( \sum_{n=1}^{\infty} b_n ) be a series with positive terms such that ( \lim_{n \to \infty} \frac{a_n}{b_n} = \rho ). If ( 0 &lt; \rho &lt; \infty ), then both series converge or both diverge.</td>
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<tr>
<td>Ratio Test</td>
<td>Let ( \sum_{n=1}^{\infty} a_n ) be a series with positive terms and suppose ( \lim_{n \to \infty} a_{n+1} ) ( a_n = \rho ).</td>
<td>Try this test when ( a_n ) involves factorials or ( n )-th powers.</td>
</tr>
<tr>
<td></td>
<td>a) Series converges if ( \rho &lt; 1 ).</td>
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<td>b) Series diverges if ( \rho &gt; 1 ).</td>
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<td></td>
<td>c) No conclusion if ( \rho = 1 ).</td>
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<tr>
<td>Root Test</td>
<td>Let ( \sum_{n=1}^{\infty} a_n ) be a series with positive terms and suppose ( \lim_{n \to \infty} \sqrt[n]{a_n} = \rho ).</td>
<td>Try this test when ( a_n ) involves ( n )th powers.</td>
</tr>
<tr>
<td></td>
<td>a) Series converges if ( \rho &lt; 1 ).</td>
<td></td>
</tr>
<tr>
<td></td>
<td>b) Series diverges if ( \rho &gt; 1 ).</td>
<td></td>
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<tr>
<td></td>
<td>c) No conclusion if ( \rho = 1 ).</td>
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<tr>
<td>Alternating Series Test</td>
<td>Let ( \sum_{n=1}^{\infty} a_n ) be a series with alternating terms if ( \lim_{n \to \infty} a_n = 0 ) and (</td>
<td>a_n</td>
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