Math 122
Quiz 8 Review

Determine if the following series converge absolutely, converge conditionally, or diverge:

1. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n+1}}$

2. $\sum_{n=1}^{\infty}(-1)^n \frac{n!}{2^n}$

3. $\sum_{n=1}^{\infty}(-1)^n \frac{\arctan n}{n^2+1}$

4. $\sum_{n=1}^{\infty}(-1)^n \left(\frac{\ln n}{\ln n^2}\right)^n$

5. $\sum_{n=1}^{\infty}(-1)^n \frac{n}{n+1}$

6. $\sum_{n=1}^{\infty}(-1)^n \frac{\sin n}{n^2}$

7. $\sum_{n=1}^{\infty}(-1)^n \frac{1}{\sqrt{n+1}+\sqrt{n}}$

8. $\sum_{n=1}^{\infty}(-1)^n \frac{(2n)!}{2^n n!\, n}$

9. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}\, n^3}{e^n}$

10. $\sum_{n=1}^{\infty} \frac{(-1)^n\, \ln n}{n}$

Find the radius and interval of convergence for:

11. $\sum_{n=1}^{\infty} 3^n x^n$

12. $\sum_{n=1}^{\infty} \frac{n!}{3^n}(x-2)^n$

13. $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n+1}$

14. $\sum_{n=1}^{\infty} \frac{1}{n^2+1}(x-3)^n$

15. $\sum_{n=1}^{\infty} \frac{9^n}{n!}(x-1)^n$

Find the n-th degree Taylor polynomial centered at $x = a$ for:

16. $f(x) = \ln x \qquad a = 1 \qquad n = 4$

17. $f(x) = \arctan x \qquad a = 0 \qquad n = 2$

18. $f(x) = \cos x \qquad a = \frac{\pi}{4} \qquad n = 3$

19. $f(x) = \frac{1}{\sqrt{1-x}} \qquad a = 0 \qquad n = 4$

Answers

1. Converge Conditionally

2. Diverge

3. Converge Absolutely

4. Converge Absolutely

5. Diverge

6. Converge Absolutely

7. Converge Conditionally

8. Diverges

9. Converge Absolutely

10. Converge Conditionally

11. $r = \frac{1}{3}$, $(-\frac{1}{3}, \frac{1}{3})$

12. $r = 0$, $x = 2$

13. $r = 1$, $[-3, -1)$

14. $r = 1$, $[2, 4]$

15. $r = \infty$, $(-\infty, \infty)$

16. $T_4(x) = (x-1) - \frac{1}{2}(x-1)^2 + \frac{1}{3}(x-1)^3 - \frac{1}{4}(x-1)^4$

17. $T_2(x) = x$

18. $T_3(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2}\left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{2!\,2}\left(x - \frac{\pi}{4}\right)^2 + \frac{\sqrt{2}}{3!\,2}\left(x - \frac{\pi}{4}\right)^3$

19. $T_4(x) = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \frac{35}{128}x^4$