Quiz Book Number

# Math 121 Test 3

## November 18, 2025

EF:

| | |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| Total | |

**Name** KEY

Directions:

1. No books, notes or playing Christmas songs before Thanksgiving. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.
2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.
3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.
4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.
5. This test has 5 problems.

1. (20 points)

(a) Find the Linear approximation of $f(x) = \sqrt[3]{x}$ at $a = 125$ and use it to approximate $\sqrt[3]{126}$

7

$L(x) = f(a) + f'(a)(x-a)$

$f(125) = \sqrt[3]{125} = 5$

$f'(x) = \frac{1}{3\sqrt[3]{x^2}}$ $\quad f'(125) = \frac{1}{3\sqrt[3]{(125)^2}} = \frac{1}{75}$

$L(x) = 5 + \frac{1}{75}(x - 125)$

$\sqrt[3]{126} = f(126) \approx L(126) = 5 + \frac{1}{75}(126-125) \approx 5.01333$

(b) Find the maximum and minimum values for $f(x) = (x^2 - 1)^{1/3}$ on the interval $[-1, 2]$

7

$f'(x) = \frac{1}{3}(x^2-1)^{-\frac{2}{3}}(2x) = \frac{2x}{3(x^2-1)^{\frac{2}{3}}}$

$2x = 0 \quad x = 0$

$x^2 - 1 = 0 \quad x = \pm 1$

| x | f(x) | |
|---|---|---|
| -1 | 0 | |
| 0 | -1 | min |
| 1 | 0 | |
| 2 | $\sqrt[3]{3}$ | max |

(c) Indicate whether the following statements are true or false by circling the appropriate letter. A statement which is sometimes true and sometimes false should be marked false.

6

i) If $f(x)$ is not continuous on $[0, 1]$ then $f(x)$ has no maximum on $[0, 1]$. **T** **F**

ii) If $f(x)$ is continuous, but has no critical points on $[0, 1]$ then $f(x)$ has no maximum on $[0, 1]$. **T** **F**

iii) If $f(x)$ has a critical point at $x = 1$, then $f(x)$ has a local minimum or maximum at $x = 1$. **T** **F**

2. (20 points) For $f(x) = 4x^{1/3} + x^{4/3}$

$$\left( \text{Hint: } f'(x) = \frac{4x+4}{3x^{2/3}} \text{ and } f''(x) = \frac{4x-8}{9x^{5/3}} \right)$$

find:

3

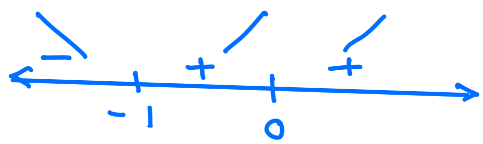
(a) Domain: ALL x

(b) Range: $[-3, \infty)$

3

(c) $x$-intercepts: $x^{\frac{1}{3}}(4+x) = 0$ $\quad x = 0, -4$

(d) $y$-intercepts: $y = 0$




5

(e) Where $y$ is increasing:

$(-1, 0) \cup (0, \infty)$

(f) Where $y$ is decreasing:

$(-\infty, -1)$

(g) Critical points:

$x = -1, 0$ $\quad$ ($x = -1$ IS A MIN)

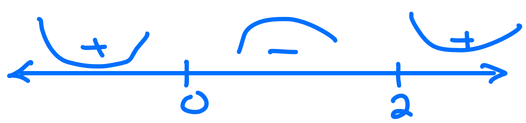
5

(h) Where $y$ is concave up:

$(-\infty, 0) \cup (2, \infty)$

(i) Where $y$ is concave down:

$(0, 2)$

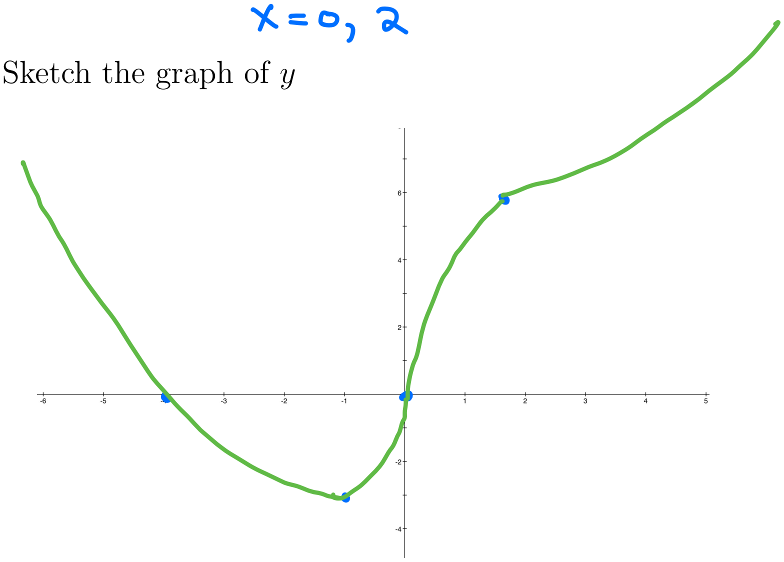


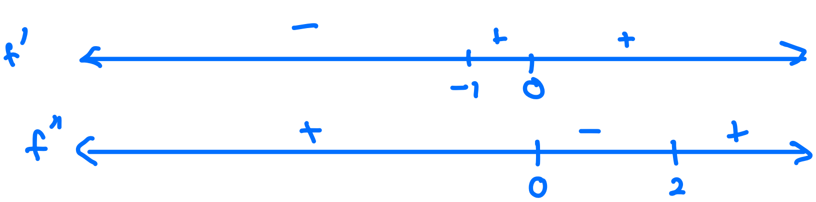

(j) Inflection points:

$x = 0, 2$

4

(k) Sketch the graph of $y$





3. (20 points)

5

(a) Compute $\lim_{x \to 0} \frac{e^{x^2} - 1}{2x^2} = \frac{0}{0}$

$$= \lim_{x \to 0} \frac{2\cancel{x} e^{x^2}}{4\cancel{x}} = \frac{2}{4} = \frac{1}{2}$$

(b) Compute $\lim_{x \to \infty} [\ln x - \ln(x+2)]$

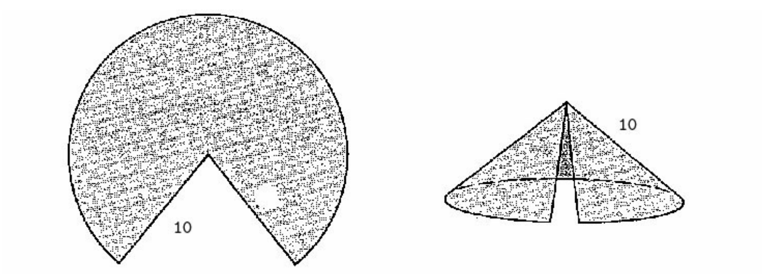
5

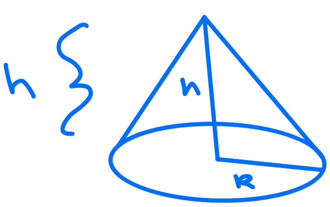
$$= \lim_{x \to \infty} \ln\left(\frac{x}{x+2}\right) = \ln\left(\lim_{x \to \infty} \frac{x}{x+2}\right)$$

$$= \ln(1) = 0$$

10

(c) A cone is formed from a circular sheet with a radius of 10 meters. A sector is removed, and the two edges of the remaining portion are joined together. What is the maximum possible volume of the resulting cone? (Hint: The volume of a cone is $V = \frac{1}{3}\pi r^2 h$.)




$$V = \frac{1}{3}\pi R^2 h$$

$$h^2 + R^2 = 100$$

$$R^2 = 100 - h^2$$

$$V = \frac{1}{3}\pi(100 - h^2)h = \frac{1}{3}\pi(100h - h^3)$$

$$\frac{dV}{dh} = \frac{1}{3}\pi(100 - 3h^2)$$

$$100 - 3h^2 = 0$$

$$h = \sqrt{\frac{100}{3}} = \frac{10\sqrt{3}}{3}$$

| $h$ | $V$ | |
|---|---|---|
| 0 | 0 | |
| 10 | 0 | |
| $\frac{10\sqrt{3}}{3}$ | $\frac{1}{3}\pi\left(\frac{200}{3}\right)\left(\frac{10\sqrt{3}}{3}\right)$ | MAX |

4. (20 points)

(a) For $f(x) = x^2 + 1$ on $[0, 3]$

5

i. Compute $R_3 = \sum_{j=1}^{3} f(a + j\Delta x)\Delta x$, the right Riemann sum with 3 sub-intervals.

$\Delta x = \frac{3-0}{3} = 1$

$x_1 = 1$, $x_2 = 2$, $x_3 = 3$

$f(x_1) = 2 \quad f(x_2) = 5 \quad f(x_3) = 10$

$R_3 = (2 + 5 + 10)(1) = 17$

5

ii. Compute $\lim_{n\to\infty} \sum_{j=1}^{n} \left[ \left(\frac{3j}{n}\right)^2 + 1 \right] \frac{3}{n}$ (Hint: See part (i) )

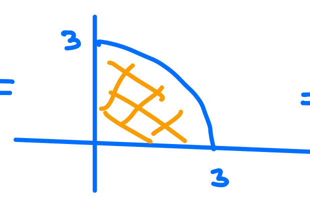
$= \int_0^3 (x^2 - 1)\, dx = \frac{x^3}{3} + x \Big|_0^3 = 9 + 3 = 12$

(b) If

5

$$F(x) = \int_0^x \sqrt{9 - t^2}\, dt$$

i. Find $F(3)$

$= \int_0^3 \sqrt{9 - t^2}\, dt =$  $= \frac{9\pi}{4}$



5

ii. Find $F'(2)$

$F'(x) = \sqrt{9 - x^2}$

$F'(2) = \sqrt{9 - 4} = \sqrt{5}$

5. (20 points)

5

(a) Compute $\int (1+x^3)^2 \, dx$

$$= \int 1 + 2x^3 + x^6 \, dx$$

$$= x + \frac{x^4}{2} + \frac{x^7}{7} + C$$

5

(b) Compute $\int x\sqrt{2-x} \, dx$

$u = 2 - x$ $\quad x = 2 - u$

$du = -dx$

$$= -\int (2-u)\sqrt{u} \, du = -\int 2\sqrt{u} - u^{\frac{3}{2}} \, du$$

$$= -\left(2 \frac{2}{3} u^{\frac{3}{2}} - \frac{2u^{\frac{5}{2}}}{5}\right) = -\left(\frac{4}{3}(2-x)^{\frac{3}{2}} - \frac{2}{5}(2-x)^{\frac{5}{2}}\right) + C$$

5

(c) Compute $\int_0^1 x\sqrt{9x^2+16} \, dx$

$u = 9x^2 + 16$ $\quad dx = \frac{du}{18x}$

$du = 18x \, dx$

$$= \int \cancel{x} \sqrt{u} \, \frac{du}{18\cancel{x}} = \frac{1}{18} \frac{2}{3} u^{\frac{3}{2}} = \frac{1}{27} (9x^2+16)^{\frac{3}{2}} \Big|_0^1$$

$$= \frac{1}{27}(125 - 64) = \frac{61}{27}$$

5

(d) Compute $\int \frac{1}{\sqrt{1-16x^2}} \, dx$

$u = 4x$ $\quad dx = \frac{du}{4}$

$du = 4 \, dx$

$$= \int \frac{1}{\sqrt{1-u^2}} \frac{du}{4} = \frac{1}{4} \text{ARCSIN}(u)$$

$$= \frac{1}{4} \text{ARCSIN}(4x) + C$$