Quiz Book Number

Math 121 Test 2

October 14, 2025

EF:

1	
2	
3	
4	
5	
Total	

Name KEY

Directions:

1. No books, notes or music parts with only one note. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.

2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.

3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.

4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

5. This test has 5 problems.

1. (20 points) Find the derivatives of the following functions:

(a) $f(x) = 3x^3 + \frac{3}{x^3}$

$$f'(x) = 9x^2 - 9x^{-4}$$

(b) $p(x) = (\sqrt[3]{x} + 3)\tan x$

$$p'(x) = (\sqrt[3]{x} + 3)\sec^2 x + \left(\frac{1}{3}x^{-\frac{2}{3}}\right)\tan x$$

(c) $q(x) = \frac{3x + e^x}{3x + \cos x}$

$$q'(x) = \frac{(3x + \cos x)(3 + e^x) - (3x + e^x)(3 - \sin x)}{(3x + \cos x)^2}$$

(d) $c(x) = \sin\left(\sqrt{x^3 + 1}\right)$

$$c'(x) = \frac{1}{2}(x^3 + 1)^{-\frac{1}{2}}(3x^2)\cos\left(\sqrt{x^3+1}\right)$$

2. (20 points) If $f(3) = 2$ and $f'(3) = -1$, find:

(a) $s'(3)$ where $s(x) = 3x^3 + 3f(x)$

$$s'(x) = 9x^2 + 3f'(x)$$

$$s'(3) = 81 + 3(-1) = 78$$

(b) $p'(3)$ where $p(x) = x^3 f(x)$

$$p'(x) = x^3 f'(x) + 3x^2 f(x)$$

$$p'(3) = (27)(-1) + 27(2) = 27$$

(c) $g'(3)$ where $g(x) = \arctan(f(x))$

$$g'(x) = \frac{1}{1+(f(x))^2} \cdot f'(x)$$

$$g'(3) = \frac{1}{1+2^2} \cdot (-1) = -\frac{1}{5}$$

(d) $h'(3)$ where $h(x) = 3^{f(x)}$

$$h'(x) = 3^{f(x)} \ln 3 \cdot f'(x)$$

$$h'(3) = 3^2 \ln 3 \ (-1)$$

$$= -9 \ln 3$$

3. (20 points) For parts (a)-(c), find $\frac{dy}{dx}$:

(a) $y = \ln[(\ln x)^3]$ OR $y = 3\ln(\ln x)$

$$\frac{dy}{dx} = \frac{1}{(\ln x)^3} \cdot 3(\ln x)^2 \cdot \frac{1}{x}$$

OR

$$\frac{dy}{dx} = 3\,\frac{1}{\ln x} \cdot \frac{1}{x}$$

(b) $y = e^{\arcsin x} + \sinh(e^x)$

$$\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \cdot e^{\text{ARCSIN}x} + e^x \cosh(e^x)$$

(c) $y = (x^2+1)^{\sin x}$

$$\ln y = \ln\left((x^2+1)^{\sin x}\right) = \sin x \ln(x^2+1)$$

$$\frac{1}{y}\frac{dy}{dx} = (\sin x)\frac{2x}{x^2+1} + \cos x \ln(x^2+1)$$

$$\frac{dy}{dx} = (x^2+1)^{\sin x}\left[\sin x\left(\frac{2x}{x^2+1}\right) + \cos x \ln(x^2+1)\right]$$

(d) Find y' and y'' for $y = x^3e^x$

$$y' = x^3e^x + 3x^2e^x = (x^3+3x^2)e^x$$

$$y'' = (x^3+3x^2)e^x + (3x^2+6x)e^x$$

4. (20 points)

(a) Find the equation of the line tangent to the curve

$$x^2 + xy - y^3 = 7$$

at the point $(3, 2)$.

$$2x + y + x\frac{dy}{dx} - 3y^2\frac{dy}{dx} = 0$$

$$6 + 2 + 3\frac{dy}{dx} - 12\frac{dy}{dx} = 0$$

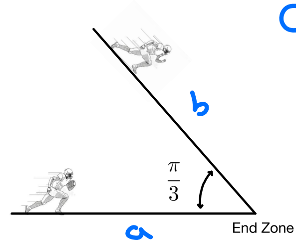
$$9\frac{dy}{dx} = 8 \qquad \frac{dy}{dx} = \frac{8}{9}$$

$$y - 2 = \frac{8}{9}(x - 3)$$

(b) Caleb is running for a touchdown at 6 yards per second. Ria is trying to catch him at 7 yards per second. The angle between them is $\frac{\pi}{3}$ (see picture). When Caleb is 8 yards from the end zone, and Ria is 5 yards from the end zone, how fast is the distance between them be changing?
(Hint: Use the law of cosines. $c^2 = a^2 + b^2 - 2ab\cos\theta$.)

$a = 8 \qquad b = 5$

$\frac{da}{dt} = -6 \qquad \frac{db}{dt} = -7$



$c^2 = 64 + 25 - 40$

$= 49$

$c = 7$

$$c^2 = a^2 + b^2 - \cancel{2}ab\,\cancel{\cos(\tfrac{\pi}{3})}$$

$$2c\frac{dc}{dt} = 2a\frac{da}{dt} + 2b\frac{db}{dt} - a\frac{db}{dt} - b\frac{da}{dt}$$

$$(2)(7)\frac{dc}{dt} = 2(8)(-6) + 2(5)(-7) - (8)(-7) - 5(-6)$$

$$(2)(7)\frac{dc}{dt} = -96 - 70 + 56 + 30 = -80$$

$$\frac{dc}{dt} = -5.7 \text{ YARD/SEC}$$

5. (20 points)

(a) Use the definition of derivative to find $f'(x)$ for

$$f(x) = \frac{x}{x-1}$$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{x+h}{x+h-1} - \frac{x}{x-1}}{h}$$

$$= \lim_{h \to 0} \frac{(x+h)(x-1) - x(x+h-1)}{h(x+h-1)(x-1)}$$

$$= \lim_{h \to 0} \frac{\cancel{x^2} + \cancel{xh} \cancel{-x} - \overset{-1}{\cancel{h}} - \cancel{x^2} - \cancel{xh} + \cancel{x}}{\cancel{h}(x+h-1)(x-1)} = \frac{-1}{(x-1)^2}$$

(b) Indicate whether the following statements are true or false by circling the appropriate letter. A statement which is sometimes true and sometimes false should be marked false.

i) If $f'(x) = f''(x)$ for all x then $f'(x) = f'''(x)$ for all x. **T** **F**

ii) If $f'(1) = 0$ then for any differentiable function $g(x)$ then $(g \circ f)'(1) = 0$ **T** **F**

iii) If $f'(2) = 0$ then for any differentiable function $g(x)$ then $(f \circ g)'(2) = 0$ **T** **F**

iv) If $f(x)$ and $g(x)$ are differentiable at $x = 3$, $h(x) = f(x)g(x)$ and $h'(3) = 0$, then $f(3)g'(3) = -f'(3)g(3)$ **T** **F**

v) If the tangent line to $y = f(x)$ at $x = 4$ and $x = 5$ is $y = -x + 5$ the $f'(4) = f'(5)$. **T** **F**