Math 121 Test 1

September 16, 2025

EF:

Name KEY

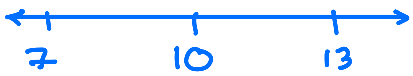
1	
2	
3	
4	
5	
Total	

Directions:

1. No books, notes or gas station sushi. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.
2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.
3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.
4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

1. (20 Points)

(a) If you were to write the interval $[7, 13]$ in the form $|x - a| \leq b$, what are a and b?

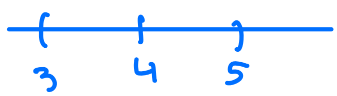


$|x - 10| \leq 3$

$a = 10$

$b = 3$

(b) If $|x - 4| \leq 1$, what is the largest possible value of $|x + 4|$?



$x \leq 5$

$|x+4| \leq |5+4| = 9$

9

(c) Find the equation of the line through $(2, 1)$ and $(9, 5)$.

$$m = \frac{5-1}{9-2} = \frac{4}{7}$$

$$y - 1 = \frac{4}{7}(x-2)$$

(d) If $f(x) = x^3 + x^2$ and $g(x) = \cos x$, find $(f \circ g)(x)$ and $(g \circ f)(x)$.

$$(f \circ g)(x) = f(g(x)) = f(\cos x) = \cos^3 x + \cos^2 x$$

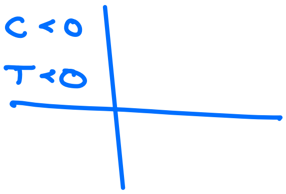
$$(g \circ f)(x) = g(f(x)) = g(x^3 + x^2) = \cos(x^3 + x^2)$$

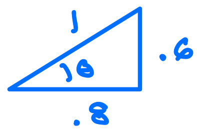
2. (20 points)

(a) Find $\cos\theta$ and $\tan\theta$ if $\sin\theta = 0.6$ and $\pi/2 < \theta < \pi$.

C < 0

T < 0





$\cos\theta = -0.8$

$\tan\theta = -0.75$

(b) Find $f^{-1}(x)$ for $f(x) = \dfrac{3x+2}{x}$

$$y = \frac{3x+2}{x}$$

$$x = \frac{3y+2}{y}$$

$$xy = 3y+2$$

$$xy - 3y = 2$$

$$y(x-3) = 2$$

$$y = \frac{2}{x-3}$$

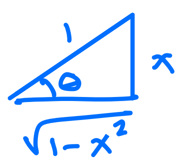
$$f^{-1}(x) = \frac{2}{x-3}$$

(c) Which of the following is equal to $\tan(\arcsin x)$?

a) $\sqrt{1-x^2}$ b) $\dfrac{\sqrt{1-x^2}}{x}$ $\boxed{\text{c) } \dfrac{x}{\sqrt{1-x^2}}}$ d) $\dfrac{x}{\sqrt{1+x^2}}$

$\theta = \arcsin x$

$\sin\theta = x$



$$\tan\theta = \frac{x}{\sqrt{1-x^2}}$$

(d) Find x if

$$\ln(x^4) - \ln(x^2) = 2$$

$$\ln(x^2) = 2$$

$$2\ln(x) = 2$$

$$\ln x = 1$$

$$x = e$$

$x = \pm e$ IS OK ALSO

3. (20 points) Find the following limits:

(a) $\lim_{x\to 1} \frac{x^2-3x+2}{x+1}$ $= \frac{1-3+2}{2} = 0$

(b) $\lim_{x\to 1} \frac{x^2-3x+2}{x-1}$ $= \lim_{x\to 1} \frac{\cancel{(x-1)}(x-2)}{\cancel{(x-1)}} = -1$

(c) $\lim_{x\to 1} \left(\frac{1}{x-1} - \frac{2}{x^2-1}\right)$ $= \lim_{x\to 1} \left(\frac{x+1}{x^2-1} - \frac{2}{x^2-1}\right)$

$= \lim_{x\to 1} \frac{\overset{1}{\cancel{x-1}}}{\cancel{(x-1)}(x+1)} = \frac{1}{2}$

(d) $\lim_{x\to 1} \frac{2-\sqrt{x+3}}{x-1}$ $\left(\frac{2+\sqrt{x+3}}{2+\sqrt{x+3}}\right)$

$= \lim_{x\to 1} \frac{4-(x+3)}{(x-1)(2+\sqrt{x+3})} = \lim_{x\to} \frac{\overset{-1}{\cancel{1-x}}}{\cancel{(x-1)}(2+\sqrt{x+3})}$

$= \frac{-1}{4}$

4. (20 points)

(a) Find $\lim_{x \to 0} \frac{x \sin x}{1 - \cos x}$ $\cdot \frac{1+\cos x}{1+\cos x} = \lim_{x\to 0} \frac{x \sin x (1+\cos x)}{1-\cos^2 x}$

$= \lim_{x\to 0} \frac{x \cancel{\sin x} (1+\cos x)}{\sin^{\cancel{2}} x} = \lim_{x\to 0} \frac{x}{\sin x} (1+\cos x)$

$= 1 \cdot 2 = 2$

(b) Find $\lim_{x \to 0} x^2 \cos\left(\frac{1}{x^2}\right)$ $= 0$

$-1 \leq \cos\left(\frac{1}{x^2}\right) \leq 1$

$-x^2 \leq \cos x \left(\frac{1}{x^2}\right) \leq x^2$

$\searrow 0 \qquad \swarrow 0$

(c) Find $\lim_{x \to -\infty} \frac{8x + 7}{\sqrt{9x^2 + 6x}}$

$= -\frac{8}{3}$

(d) Find a and b so that $f(x)$ is continuous

$$f(x) = \begin{cases} ax^2 + b & x < 1 \\ 3 & x = 1 \\ 2x + a & x > 1 \end{cases}$$

$\lim_{x\to 1^-} f(x) = a+b=3$

$\lim_{x\to 1^+} f(x) = 2+a=3$

$a+b=3$

$2+a=3$

$a=1$

$b=2$

5. (20 points) Indicate whether the following statements are true or false by circling the appropriate letter. A statement which is sometimes true and sometimes false should be marked false.

a) If $\lim_{x \to c} f(x) = 1$, then $f(c) = 1$. **T** **F**

b) If $\lim_{x \to c} f(x) = 2$, then $\lim_{x \to c^-} f(x) = \lim_{x \to c^+} f(x)$ **T** **F**

c) If $f(x)$ is not continuous at $x = 3$, then $\lim_{x \to 3} f(x)$ does not exist. **T** **F**

d) If $f(x)$ and $g(x)$ are inverse functions and $f(2) = 1$ then $g(1) = 2$ **T** **F**

e) If $f(1) = -2$ and $f(4) = 5$, then there is a c with $1 < c < 4$ where $f(c) = 0$ **T** **F**

f) If $\lim_{x \to 2} f(x) = 6$ and $\lim_{x \to 2} g(x) = 4$ then $\lim_{x \to 2} \dfrac{x\, f(x)}{g(x)} = 3$ **T** **F**

g) If $\lim_{x \to 1} f(x) = 2$, there is a $\delta > 0$ such that if $0 < |x - 1| < \delta$, then $|f(x) - 2| < 0.001$. **T** **F**

h) If $\lim_{x \to 1} f(x) = 2$, there is a $\delta > 0$ such that if $0 < |x - 1| < \delta$, then $f(x) = 2$. **T** **F**

i) If $\lim_{x \to 1} f(x) = 2$, there is a $\epsilon > 0$ such that if $|f(x) - 2| < \epsilon$ then x must be 1. **T** **F**

j) $1 + 1 = 2$ **T** **F**