

Math 122
Quiz 9 Review

Determine if the following series converge absolutely, converge conditionally, or diverge:

1. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n+1}}$

2. $\sum_{n=1}^{\infty} (-1)^n \frac{n!}{2^n}$

3. $\sum_{n=1}^{\infty} (-1)^n \frac{\arctan n}{n^2 + 1}$

4. $\sum_{n=1}^{\infty} (-1)^n \left(\frac{\ln n}{\ln n^2} \right)^n$

5. $\sum_{n=1}^{\infty} (-1)^n \frac{n}{n+1}$

6. $\sum_{n=1}^{\infty} (-1)^n \frac{\sin n}{n^2}$

7. $\sum_{n=1}^{\infty} (-1)^n \frac{1}{\sqrt{n+1} + \sqrt{n}}$

8. $\sum_{n=1}^{\infty} (-1)^n \frac{(2n)!}{2^n n! n}$

9. $\sum_{n=1}^{\infty} \frac{(-1)^{n+1} n^3}{e^n}$

10. $\sum_{n=1}^{\infty} \frac{(-1)^n \ln n}{n}$

Find the radius and interval of convergence for:

11. $\sum_{n=1}^{\infty} 3^n x^n$

12. $\sum_{n=1}^{\infty} \frac{n!}{3^n} (x-2)^n$

13. $\sum_{n=1}^{\infty} \frac{(x+2)^n}{n+1}$

14. $\sum_{n=1}^{\infty} \frac{1}{n^2 + 1} (x-3)^n$

15. $\sum_{n=1}^{\infty} \frac{9^n}{n!} (x-1)^n$

Find the n -th degree Taylor polynomial centered at $x = a$ for:

16. $f(x) = \ln x \quad a = 1 \quad n = 4$

17. $f(x) = \arctan x \quad a = 0 \quad n = 2$

18. $f(x) = \cos x \quad a = \frac{\pi}{4} \quad n = 3$

19. $f(x) = \frac{1}{\sqrt{1-x}} \quad a = 0 \quad n = 4$

Answers

1. Converge Conditionally
2. Diverge
3. Converge Absolutely
4. Converge Absolutely
5. Diverge
6. Converge Absolutely
7. Converge Conditionally
8. Diverges
9. Converge Absolutely
10. Converge Conditionally
11. $r = \frac{1}{3}, (-\frac{1}{3}, \frac{1}{3})$
12. $r = 0, x = 2$
13. $r = 1, [-3, -1)$
14. $r = 1, [2, 4]$
15. $r = \infty, (-\infty, \infty)$
16. $T_4(x) = (x - 1) - \frac{1}{2}(x - 1)^2 + \frac{1}{3}(x - 1)^3 - \frac{1}{4}(x - 1)^4$
17. $T_2(x) = x$
18. $T_3(x) = \frac{\sqrt{2}}{2} - \frac{\sqrt{2}}{2} \left(x - \frac{\pi}{4}\right) - \frac{\sqrt{2}}{2! \cdot 2} \left(x - \frac{\pi}{4}\right)^2 + \frac{\sqrt{2}}{3! \cdot 2} \left(x - \frac{\pi}{4}\right)^3$
19. $T_4(x) = 1 + \frac{1}{2}x + \frac{3}{8}x^2 + \frac{5}{16}x^3 + \frac{35}{128}x^4$