

# Math 122

## Test 2 - Review 1

### I. Applications of Integration

#### A) Probability

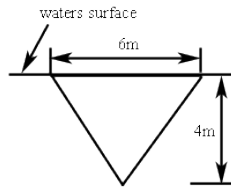
- Determine the value of  $C$  so that  $p(x)$  is a probability density function on the given interval.
  - $p(x) = C(6 - x)$  for  $1 \leq x \leq 3$
  - $p(x) = Ce^{-3x}$  for  $x \geq 1$
  - $p(x) = Cx^3(1 - x)$  for  $0 \leq x \leq 1$
- Let  $p(x) = 4x^3$  for  $0 \leq x \leq 1$ .
  - Find  $P(0 \leq X \leq 1/2)$
  - Find  $P(X \geq 3/4)$
- Let  $p(x) = \frac{12x^2}{625}(5 - x)$  for  $0 \leq x \leq 5$ .
  - Find  $P(1 \leq X \leq 4)$
  - Find  $P(X \geq 3)$
- Find the value of  $C$  so that  $p(x) = \frac{C}{x^2 + 9}$  for the interval  $-\infty < x < \infty$  is a probability density function?
- Find the value of  $c$  so that  $p(x) = \frac{x^4}{625}$  for the interval  $0 < x < c$  is a probability density function?
- A random variable  $X$  has probability density function  $p(x) = \frac{3}{5}(4 - x^2)$  for  $1 \leq x \leq 2$ , find the expected value  $\mu$ .
- A random variable  $X$  has probability density function  $p(x) = (24 - 2x)/45$  for  $3 \leq x \leq 6$ .
  - Find the expected value  $\mu$ .
  - Determine the value of  $k$  so that  $P(X > k) = \frac{28}{45}$

#### B) Arc Length and Surface Area

- Find the arc length of  $y = \frac{2}{3}x^{3/2} + 1$  for  $0 \leq x \leq 3$ .
- Find the arc length of  $y = \ln(\sec x)$  for  $0 \leq x \leq \pi/4$ .
- Find the arc length of  $y = \frac{2}{3}(x - 1)^{3/2}$  for  $1 \leq x \leq 4$ .
- Find the arc length of  $y = \frac{x^3}{6} + \frac{1}{2x}$  for  $1 \leq x \leq 4$ .
- Find the surface area when  $y = \sqrt{x}$  for  $2 \leq x \leq 6$  is rotated about the  $x$ -axis.
- Find the surface area when  $y = \sqrt{7 - x}$  for  $0 \leq x \leq 3$  is rotated about the  $x$ -axis.

### C) Fluid Force

14. Find the fluid force on the front of an aquarium that is a 2 foot by 3 foot rectangle filled with water ( $\rho = 62.5 \text{ lb/ft}^3$ )
15. Find the fluid force on a vertical rectangle plate, 4 meters long, 2 meters high and the top edge is 1 meter below the surface of the water ( $\rho = 9810 \text{ N/m}^3$ ).
16. Determine the force on the following triangular plate that is submerged in water ( $\rho = 9810 \text{ N/m}^3$ ) as shown.



17. Find the force on a circular plate of radius 2 meters that is submerged 6 meters in the water ( $\rho = 9810 \text{ N/m}^3$ ).

### D) Center of Mass

18. Find the center of mass of the region bounded by  $y = 2 \sin(2x)$  and  $y = 0$  for  $0 \leq x \leq \pi/2$ .
19. Find the center of mass of the region bounded by  $y = x^3$  and  $y = \sqrt{x}$ .
20. Find the center of mass of the region bounded by  $y = 9 - x^2$  and  $y = 0$  for  $0 \leq x \leq 3$ .
21. Find the center of mass of the region bounded by  $y = 9 - x^2$  and  $y = \frac{5}{2}x$  for  $0 \leq x \leq 2$ .

## II. Differential Equations

### A) Separable

Solve the following differential equations:

- $y' = 3(y + 7)x^2$
- $y' = \frac{x}{3y^2}$
- $y' = \frac{4 - 2x}{3y^2 - 5}; \quad y(1) = 3$
- $y' = 2y - 3; \quad y(0) = 2$
- $y' = y + 1; \quad y(0) = 1$
- $y' = 3x^2y^2; \quad y(0) = 1$

**B)**  $y' = k(y - b)$

7. Solve  $y' = 3(y - 6)$       $y(0) = 4$

8. An object with temperature of  $150^\circ$  is placed in a freezer whose temperature is  $30^\circ$ . If the object has cooled to  $120^\circ$  after 8 minutes, what will its temperature be after 16 minutes?

9. A cup of coffee is poured from a pot whose contents are  $95^\circ\text{C}$  into a non-insulated cup in a room at  $20^\circ\text{C}$ . After a minute, the coffee has cooled to  $90^\circ\text{C}$ . How much time is required before the coffee reaches a drinkable temperature of  $65^\circ\text{C}$ ?

10. A pot of liquid is put on the stove to boil. The temperature of the liquid reaches  $170^\circ\text{F}$  and then the pot is taken off the burner and placed on a counter in the kitchen. The temperature of the air in the kitchen is  $76^\circ\text{F}$ . After two minutes the temperature of the liquid in the pot is  $123^\circ\text{F}$ . How long before the temperature of the liquid in the pot will be  $84^\circ\text{F}$ ?

11. A pan of water at 46 degrees was put into a fridge. Ten minutes later, its temperature was 39. Ten minutes after that, it was 33. How cold was the fridge?

### **C) Slope Fields**

Sketch the slope field and a likely solution curve for:

12.  $\frac{dy}{dx} = x^2 + y^2$       $y(0) = 1$

13.  $\frac{dy}{dx} = 2x - 3y$       $y(0) = 1$

## D) Euler's Method

Use Euler's method with  $h = 0.1$  to approximate  $y(1)$  for the initial value problem:

14.  $\frac{dy}{dx} = x^2 + y^2$       $y(0) = 1$

15.  $\frac{dy}{dx} = 2x - 3y$       $y(0) = 1$

## E) Logistic Differential Equation

16. If  $\frac{dy}{dt} = y \left(1 - \frac{y}{5}\right)$       $y(0) = 1$ , find  $y(t)$

17. If  $\frac{dy}{dt} = .08y \left(1 - \frac{y}{1000}\right)$       $y(0) = 100$   
a. Find  $y(40)$

b. Find  $t$  where  $y(t) = 900$ .

18. Biologists stock a lake with 500 fish and the carrying capacity of the lake is 10,000. The number of fish triple during the first year. If the population of fish follow the logistic differential equation, how long will it take for the population of fish  $t$  reach 4,000?

19. A rumor spreads through a population of 5,000 people. Suppose 100 people initiate the rumor and 500 people have heard the rumor after 2 days. If the number of people that have heard the rumor follows the logistic differential equation, how long will it take for half the people to hear the rumor?

## F) Linear

Solve the following differential equations:

20.  $y' + (\cot x)y = 5e^{\cos x}$

25.  $y' + (\cot x)y = 3 \sin x \cos x$

21.  $x^3y' + (2 - 3x^2)y = x^3$

26.  $y' + 2y = 2x$

22.  $y' + \frac{3}{x}y = \frac{\sin x}{x^3}$

27.  $y' + \frac{1}{x}y = \sin x$

23.  $xy' + 2y = 4x^2$

28.  $x y' = x - y \quad y(2) = 1$

24.  $x(\ln x)y' + y = 2 \ln x$

29.  $y' - \frac{2y}{x+1} = (x^2 + 2x + 1)^2 \quad y(0) = 1$

## G) Applications

30. A tank contains 20 kg of salt dissolved in 5000 L of water. Brine that contains 0.03 kg of salt per liter of water enters the tank at a rate of 25 L/min. The solution is kept thoroughly mixed and drains from the tank at the same rate. How much salt remains in the tank after half an hour?

31. A tank initially contains 100 gal of a solution that holds 40 lb of a chemical. A solution containing 2 lb/gal of the chemical runs into the tank at the rate of 2 gal/min and the mixture runs out at the rate of 2 gal/min. How much chemical is in the tank after 50 min?

32. A tank initially contains 100 gal of a solution that holds 30 lb of a chemical. Water runs into the tank at the rate of 2 gal/min and the solution runs out at the same rate. How much of the chemical remains in the tank after 20 min?

33. You are making High-Sea fruit juice. The bottle says that it contains at least 2% real fruit juice. Unfortunately, one of your employees made a vat of 500 gallons of juice that is 20% real fruit juice. You decide that you will pump water into the tank and bottle the overflow (the tank only holds 500 gallons). If the water is pumped in at 20 gallons per minute, how long can you continue to bottle the juice (it must be 2% real fruit juice.)

## Answers

1. a.  $1/8$

b.  $3e^3$

c. 20

2. a.  $\frac{1}{16}$

b.  $175/256$

3. a.  $99/125$

b.  $328/625$

4.  $\frac{3}{\pi}$

5. 5

6. 1.35

7.  $\frac{22}{5}$

b. 4

8.  $\frac{14}{3}$

9.  $\ln(\sqrt{2} + 1)$

10.  $\frac{14}{3}$

11.  $\frac{87}{8}$

12.  $\frac{49\pi}{3}$

13.  $\frac{\pi}{6}(29^{3/2} - 17^{3/2})$

14. 375 pounds

15. 156,960 N

16. 156,960 N

17.  $313,920 \pi$  N

18.  $(\frac{\pi}{4}, \frac{\pi}{4})$

19.  $(\frac{12}{25}, \frac{3}{7})$

20.  $(\frac{9}{8}, \frac{18}{5})$

21.  $(\frac{22}{31}, \frac{778}{155})$

2.  $y = \sqrt[3]{\frac{x^2}{2}} + C$

3.  $y^3 - 5y = 4x - x^2 + 9$

4.  $y = \frac{1}{2}(e^{2x} + 3)$

5.  $y = 2e^x - 1$

6.  $y = \frac{1}{1 - x^3}$

7.  $y = 6 - 2e^{3t}$

8.  $97.5^\circ$

9. 7.4 minutes

10. 7.11 minutes

11. -3

14.  $y(1) \approx 7.18$

15.  $y(1) \approx 0.478$

16.  $y(t) = \frac{5e^t}{4 + e^t}$

17. a. 731.6

b.  $t = 54.9$

18. 2.0986 years

19. 4.58 days

20.  $y = \frac{1}{\sin x} [-5e^{\cos x} + C]$

21.  $y = \frac{x^3}{2} + Cx^3e^{1/x^2}$

22.  $y = \frac{1}{x^3} [-\cos x + C]$

23.  $y = x^2 + \text{Frac}Cx^2$

24.  $y = \frac{1}{\ln x} [(\ln x)^2 + C]$

25.  $y = \sin^2 x + \frac{C}{\sin x}$

26.  $y = -\frac{1}{2} + x + Ce^{-2x}$

27.  $y = \frac{1}{x} [-x \cos x + \sin x + C]$

28.  $y = \frac{x}{2}$

29.  $y = \frac{(x+1)^5 + 2(x+1)^2}{3}$

30. 38.11 kg.

31. 141.1 lbs.

32. 20.11 lbs.

33. 57 minutes

## Section II.

1.  $y = -7 + Ce^{x^3}$