Math 121 Test 1 - A

EF:

September 19, 2017

1	
2	
3	
4	
5	
6	
7	
8	
Total	

Name KEY

Directions:

1. No books, notes or evacuations. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.
2. You should show your work and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.
3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.
4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.
5. This test has 8 problems.

A

1. (10 points) Select (a) - (f) that matches each inequality.

f	$a > 3$	a) a is either to the left of -5 or the right of 5.
c	$\|a - 5\| < \frac{1}{3}$	b) a lies between 1 and 7.
e	$\|a - \frac{1}{3}\| < 5$	c) The distance from a to 5 is less than $\frac{1}{3}$
a	$\|a\| > 5$	d) The distance from a to 3 is a most 2.
b	$\|a - 4\| < 3$	e) a is less than 5 units from $\frac{1}{3}$.
d	$1 \leq a \leq 5$	f) a lies to the right of 3.

2. (10 Points) If $f(x)$ has domain $[4, 8]$ and range $[2, 6]$, find the domain and range of

   (a) $y = f(x) + 3$

   DOMAIN : [4, 8]
   RANGE : [5, 9]

   (b) $y = f(x + 3)$

   DOMAIN : [1, 5]
   RANGE : [2, 6]

   (c) $y = 3f(x)$

   DOMAIN : [4, 8]
   RANGE : [6, 18]

3. (15 points)

(a) Find the equation of the line perpendicular to $3x + 5y = 9$ and passing through $(2, 3)$.

$$5y = -3x + 9$$

$$y = -\frac{3}{5}x + \frac{9}{5} \qquad m = -\frac{3}{5}$$

$$m_{\perp} = \frac{5}{3} \qquad \boxed{y - 3 = \frac{5}{3}(x-2)}$$

(b) Complete the square and find the maximum (or minimum) of

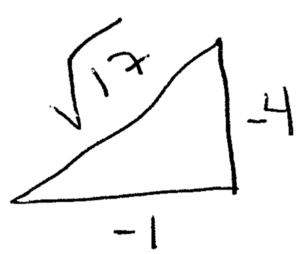
$$y = x^2 + 2x + 5$$

$$y = x^2 + 2x + 1 + 4$$

$$y = (x+1)^2 + 4$$

MIN OF 4 (AT $x = -1$)

(c) If $\tan\theta = 4$ and $\sin\theta < 0$, find $\cos\theta$ and $\sin\theta$



$$\cos\theta = \frac{-1}{\sqrt{17}}$$

$$\sin\theta = \frac{-4}{\sqrt{17}}$$

4. (10 points)

(a) Solve for x: $25^{2x-1} = 125^{3x+4}$

$$(5^2)^{2x-1} = (5^3)^{3x+4}$$

$$5^{4x-2} = 5^{9x+12}$$

$$4x-2 = 9x+12$$

$$-14 = 5x$$

$$x = \frac{-14}{5}$$

(b) Solve for x: $\log_2(5+2x) - \log_2(4-x) = 3$

$$\mathrm{LOG}_2\left(\frac{5+2x}{4-x}\right) = 3$$

$$\frac{5+2x}{4-x} = 8$$

$$5+2x = 32-8x$$

$$10x = 27$$

$$x = \frac{27}{10} = 2.7$$

5. (15 points) Compute

(a) $\lim_{x\to 4} \frac{x^2-5x+4}{x^2-4}$

$$= \lim_{x\to 4} \frac{(x-4)(x-1)}{(x-2)(x+2)} = 0$$

(b) $\lim_{x\to 0} \frac{4^{2x}-1}{4^x-1}$

$$\lim_{x\to 0} \frac{\cancel{(4^x-1)}(4^x+1)}{\cancel{(4^x-1)}} = 2$$

(c) $\lim_{x\to a} \frac{\sqrt{x}-\sqrt{a}}{x-a}$ $\quad \frac{\sqrt{x}+\sqrt{a}}{\sqrt{x}+\sqrt{a}}$

$$= \lim_{x\to a} \frac{\cancel{(x-a)}}{\cancel{(x-a)}(\sqrt{x}+\sqrt{a})} = \frac{1}{2\sqrt{a}}$$

6. (15 points) Compute

(a) $\lim_{x \to -3} \frac{\sin(x+3)}{x^2 + 8x + 15}$

$$= \lim_{x \to -3} \frac{\sin(x+3)}{(x+3)(x+5)}$$

$$= \lim_{x \to -3} \frac{\sin(x+3)}{x+3} \cdot \frac{1}{x+5} = 1 \cdot \frac{1}{2} = \frac{1}{2}$$

(b) $\lim_{x \to 2^-} \frac{x-2}{\sqrt{x^2 - 4x + 4}}$

$$= \lim_{x \to 2^-} \frac{x-2}{\sqrt{(x-2)^2}} = \lim_{x \to 2^-} \frac{x-2}{|x-2|}$$

$$= -1$$

(c) $\lim_{x \to 0} (\tan x)(\cos(\sin \frac{1}{x})) = 0$

$$-1 \leq \cos\left(\sin\left(\tfrac{1}{x}\right)\right) \leq 1$$

$$-\tan x \leq \tan x \left(\cos\left(\sin\left(\tfrac{1}{x}\right)\right)\right) \leq \tan x$$

$$\searrow 0 \qquad\qquad \swarrow 0$$

7. (15 points)

(a) Find the value of a and b so that $f(x)$ is continuous if

$$f(x) = \begin{cases} ax - b & \text{for } x \leq -1 \\ 2x^2 + 3ax + b & \text{for } -1 < x \leq 1 \\ 4 & \text{for } x > 1 \end{cases}$$

$\lim_{x \to -1^-} f(x) = -a - b$ $\lim_{x \to -1^+} f(x) = 2 - 3a + b$

$\lim_{x \to 1^-} f(x) = 2 + 3a + b$ $\lim_{x \to 1^+} f(x) = 4$

$-a - b = 2 - 3a + b$

$2a - 2b = 2$

$2 + 3a + b = 4$

$a - b = 1$

$3a + b = 2$

$4a = 3$

$\boxed{a = \frac{3}{4} \quad b = \frac{-1}{4}}$

(b) Use the I.V.T. to show that

$$2^x = bx$$

has a solution if $b > 2$.

$f(x) = 2^x - bx$ $f(x)$ IS CONT.

$f(0) = 1$

$f(1) = 2 - b < 0$ (SINCE $b > 2$)

BY THE IVT THERE IS

A $0 < C < 1$ $f(C) = 0$

$2^C = bC$

A

8. (10 points) Given that $\lim_{x \to 0} f(x) = 1$, indicate whether the following statements are true or false by circling the appropriate letter. A statement which is sometimes true and sometimes false should be marked false.

a) If $|f(x) - 1|$ is very small, then x is close to 0. **T** **F**

b) There is an $\epsilon > 0$ such that if $0 < |f(x) - 1| < \epsilon$, then $|x| < 10^{-5}$ **T** **F**

c) There is a $\delta > 0$ such that if $0 < |x| < \delta$ then $|f(x) - 1| < 10^{-5}$ **T** **F**

d) There is a $\delta > 0$ such that if $0 < |x - 1| < \delta$ then $|f(x)| < 10^{-5}$ **T** **F**

e) If $f(0) = 1$ then $f(x)$ is continuous at $x = 0$ **T** **F**