Math 122
Quiz 10 Review

1. A vector $\overrightarrow{v}$ has initial point $(-4,-1)$ and terminal point $(-2,-5)$. Find $\overrightarrow{v}$.

2. A vector $\overrightarrow{v}$ has initial point $(1,8)$ and terminal point $(3,-7)$. Find its magitude.

3. Given $\overrightarrow{w} = \overrightarrow{i}$ and $\overrightarrow{u} = 4\overrightarrow{i} - 2\overrightarrow{j}$, find $\overrightarrow{v} = 3\overrightarrow{w} + 5\overrightarrow{u}$

4. Determine which vector is parallel to the vector $\overrightarrow{v} = \langle 2,-3,-1\rangle$
a) $\langle 4,6,-2\rangle$ b) $\langle -\frac{2}{3},1,\frac{1}{3}\rangle$ c) $\langle 1,-\frac{3}{2},\frac{1}{2}\rangle$ d) $\langle 6,-9,3\rangle$ e) None of these

5. Find the center and radius of the sphere given by:

$$x^2 + y^2 + z^2 + 2x - 2y + 6z + 7 = 0$$

6. Find the center and radius of the sphere

$$x^2 + y^2 + z^2 - 4x + 8y - 10z + 20 = 0$$

7. Is the origin inside or outside the sphere

$$x^2 - 2x + y^2 + 4y + z^2 - 6z = 2$$

8. Find the parametric equations of the line through the points $(2, 0, 3)$ and $(4, 3, 3)$

9. Find the parametric equations of the line through the points $(-3, 2, 0)$ and $(4, 3, 3)$

10. Find numbers x and y such that the point $(x, y, 1)$ lies on the line passing through the points $(2, 5, 7)$ and $(0, 3, 2)$

11. Find the point where the line through $(3, 2, 4)$ with direction vector $\vec{v} = \langle 7, 5, -4 \rangle$ intersects the xy-plane.

12. Determine whether the lines intersect, are parallel, or skew:

$$L_1 : x = 4t + 2, \qquad y = 3, \qquad z = -t + 1$$
$$L_2 : x = 2t + 2, \qquad y = 2t + 3, \qquad z = t + 1$$

13. Determine whether the lines intersect, are parallel, or skew:

$$L_1 : x = 1 - 4t, \qquad y = 2 + 3t, \qquad z = 4 - 2t$$
$$L_2 : x = 2 - t, \qquad y = 1 + t, \qquad z = 2 + 6t$$

14. For vectors $\overrightarrow{a} = \langle 1, 2, -1 \rangle$ and $\overrightarrow{b} = \langle 3, 5, -1 \rangle$ find:

a. Find $2\overrightarrow{a} - 5\overrightarrow{b}$

b. $\overrightarrow{a} \cdot \overrightarrow{b}$

c. The $\cos\theta$ where θ is the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.

d. A unit vector in the direction of $\overrightarrow{a}$

e. A vector of length 3 in the direction of $\overrightarrow{a}$

15. For vectors $\overrightarrow{a} = \langle 1, 1, -2 \rangle$ and $\overrightarrow{b} = \langle 1, 2, -1 \rangle$ find:

a. Find $4\overrightarrow{a} - 3\overrightarrow{b}$

b. $\overrightarrow{a} \cdot \overrightarrow{b}$

c. The $\cos\theta$ where θ is the angle between $\overrightarrow{a}$ and $\overrightarrow{b}$.

d. A unit vector in the direction of $\overrightarrow{a}$

e. A vector of length 4 in the direction of $\overrightarrow{a}$

16. Find the value of x so that $\overrightarrow{c} = \langle 2, x, -3 \rangle$ and $\overrightarrow{d} = \langle -1, 3, -2 \rangle$ are perpendicular.

17. Let $\overrightarrow{a} = \langle 3, -1, 2 \rangle$ and $\overrightarrow{b} = \langle -2, -3, 2 \rangle$. Calculate $\text{proj}_{\overrightarrow{b}} \overrightarrow{a}$

18. Let $\overrightarrow{a} = \langle 3, -1, 2 \rangle$ and $\overrightarrow{b} = \langle -2, -3, 2 \rangle$. Calculate $\text{proj}_{\overrightarrow{a}} \overrightarrow{b}$

Answers

1. $\overrightarrow{v} = \langle 2, -4 \rangle$

2. $|\overrightarrow{v}| = \sqrt{229}$

3. $\overrightarrow{v} = 23\overrightarrow{\imath} - 10\overrightarrow{\jmath}$

4. b

5. Center: $(-1, 1, -3)$, Radius: 2

6. Center: $(2, -4, 5)$, radius 5

7. Inside

8. $x = 2 + 2t \qquad y = 3t \qquad z = 3$

9. $x = 4 + 7t \qquad y = 3 + t \qquad z = 3 + 3t$

10. $x = -\frac{2}{5}$ and $y = \frac{13}{5}$

11. $(10, 7, 0)$

12. They intersect at $(2, 3, 1)$.

13. The lines are skew.

14. **a.** $\langle -13, -21, 3 \rangle$

b. 14

c. $\cos\theta = \frac{14}{\sqrt{6}\sqrt{35}}$

d. $\frac{\overrightarrow{a}}{||\overrightarrow{a}||} = \langle \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}} \rangle$

e. $\langle \frac{3}{\sqrt{6}}, \frac{6}{\sqrt{6}}, \frac{-3}{\sqrt{6}} \rangle$

15. **a.** $\langle 1, -2, -5 \rangle$

b. 5

c. $\cos\theta = \frac{\overrightarrow{a} \cdot \overrightarrow{b}}{||\overrightarrow{a}||\,||\overrightarrow{b}||} = \frac{5}{6}$

d. $\frac{\langle 1, 1, -2 \rangle}{\sqrt{6}}$

e. $\frac{\langle 4, 4, -8 \rangle}{\sqrt{6}}$

16. $x = -\frac{4}{3}$

17. $\langle \frac{-2}{17}, \frac{-3}{17}, \frac{2}{17} \rangle$

18. $\langle \frac{3}{14}, \frac{-1}{14}, \frac{2}{14} \rangle$