

Math 122
Quiz 11 Review

1. Find the center and radius of the sphere

$$x^2 + y^2 + z^2 - 4x + 8y - 10z + 20 = 0$$

2. Is the origin inside or outside the sphere

$$x^2 - 2x + y^2 + 4y + z^2 - 6z = 2$$

3. Determine which vector is parallel to the vector $\vec{a} = \langle 2, -3, -1 \rangle$

a) $\langle 4, 6, -2 \rangle$ b) $\langle -\frac{2}{3}, 1, \frac{1}{3} \rangle$ c) $\langle 1, -\frac{3}{2}, \frac{1}{2} \rangle$ d) $\langle 6, -9, 3 \rangle$ e) None of these

4. Find the parametric equations of the line through the points $(2, 0, 3)$ and $(4, 3, 3)$

5. Find the parametric equations of the line through the points $(-3, 2, 0)$ and $(4, 3, 3)$

6. Find numbers x and y such that the point $(x, y, 1)$ lies on the line passing through the points $(2, 5, 7)$ and $(0, 3, 2)$

7. Find the point where the line through $(3, 2, 4)$ with direction vector $\vec{v} = \langle 7, 5, -4 \rangle$ intersects the xy -plane.

8. Determine whether the lines intersect, are parallel, or skew:

$$\begin{aligned} L_1 : x &= 4t + 2, & y &= 3, & z &= -t + 1 \\ L_2 : x &= 2t + 2, & y &= 2t + 3, & z &= t + 1 \end{aligned}$$

9. Determine whether the lines intersect, are parallel, or skew:

$$\begin{aligned} L_1 : x &= 1 - 4t, & y &= 2 + 3t, & z &= 4 - 2t \\ L_2 : x &= 2 - t, & y &= 1 + t, & z &= 2 + 6t \end{aligned}$$

10. For vectors $\vec{a} = \langle 1, 2, -1 \rangle$ and $\vec{b} = \langle 3, 5, -1 \rangle$ find:

- a. Find $2\vec{a} - 5\vec{b}$
- b. $\vec{a} \cdot \vec{b}$
- c. The $\cos \theta$ where θ is the angle between \vec{a} and \vec{b} .
- d. A unit vector in the direction of \vec{a}
- e. A vector of length 3 in the direction of \vec{a}

11. For vectors $\vec{a} = \langle 1, 1, -2 \rangle$ and $\vec{b} = \langle 1, 2, -1 \rangle$ find:

- a. Find $4\vec{a} - 3\vec{b}$
- b. $\vec{a} \cdot \vec{b}$
- c. The $\cos \theta$ where θ is the angle between \vec{a} and \vec{b} .
- d. A unit vector in the direction of \vec{a}
- e. A vector of length 4 in the direction of \vec{a}

12. Find the value of x so that $\vec{c} = \langle 2, x, -3 \rangle$ and $\vec{d} = \langle -1, 3, -2 \rangle$ are perpendicular.

13. Let $\vec{a} = \langle 3, -1, 2 \rangle$ and $\vec{b} = \langle -2, -3, 2 \rangle$. Calculate $\text{proj}_{\vec{b}} \vec{a}$

14. Let $\vec{a} = \langle 3, -1, 2 \rangle$ and $\vec{b} = \langle -2, -3, 2 \rangle$. Calculate $\text{proj}_{\vec{a}} \vec{b}$

Answers

- Center: $(2, -4, 5)$, radius 5
- Inside
- b
- $x = 2 + 2t$ $y = 3t$ $z = 3$
- $x = 4 + 7t$ $y = 3 + t$ $z = 3 + 3t$
- $x = -\frac{2}{5}$ and $y = \frac{13}{5}$
- $(10, 7, 0)$
- They intersect at $(2, 3, 1)$.
- The lines are skew.
- a. $\langle -13, -21, 3 \rangle$
b. 14
c. $\cos \theta = \frac{14}{\sqrt{6}\sqrt{35}}$
d. $\frac{\vec{a}}{\|\vec{a}\|} = \langle \frac{1}{\sqrt{6}}, \frac{2}{\sqrt{6}}, \frac{-1}{\sqrt{6}} \rangle$
e. $\langle \frac{3}{\sqrt{6}}, \frac{6}{\sqrt{6}}, \frac{-3}{\sqrt{6}} \rangle$
- a. $\langle 1, -2, -5 \rangle$
b. 5
c. $\cos \theta = \frac{\vec{a} \cdot \vec{b}}{\|\vec{a}\| \|\vec{b}\|} = \frac{5}{6}$
d. $\frac{\langle 1, 1, -2 \rangle}{\sqrt{6}}$
e. $\frac{\langle 4, 4, -8 \rangle}{\sqrt{6}}$
- $x = -\frac{4}{3}$
- $\langle \frac{-2}{17}, \frac{-3}{17}, \frac{2}{17} \rangle$
- $\langle \frac{3}{14}, \frac{-1}{14}, \frac{2}{14} \rangle$