Math 121 Test 1

EF:

September 18, 2012

1	
2	
3	
4	
5	
6	
7	
Total	

Name KEY

Directions:

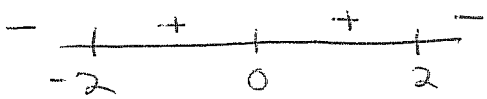
1. No books, notes or replacement referees. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.
2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.
3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.
4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.
5. This test has 7 problems.

1. (20 Points)

(a) Find the domain of $f(x) = \dfrac{\sqrt{4-x^2}}{x}$

$4 - x^2 \geq 0$ $\qquad$ $[-2, 0) \cup (0, 2]$

$x \neq 0$



(b) Find the slope, y-intercept, and x-intercept of the line with equation $y = 4 - x$

$y = -x + 4$

SLOPE $= -1$

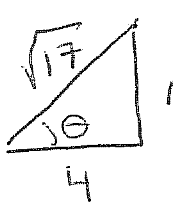
Y-INT $\Rightarrow$ $Y = 4$

X-INT $\Rightarrow$ $X = 4$

(c) Find $(f \circ g)(x)$ for $f(x) = \dfrac{1}{1+x^2}$ and $g(x) = \sin x$

$$(f \circ g)(x) = f(g(x)) = \frac{1}{1 + \sin^2 x}$$

(d) Find $\cos\theta$ if $\cot\theta = 4$ and $0 \leq \theta < \pi/2$.



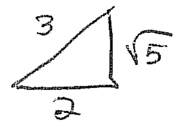
$$\cos\theta = \frac{4}{\sqrt{17}}$$

2. (20 points)

(a) Find the exact value of $\tan(\arccos \frac{2}{3})$

$\Theta = \text{ARCCOS } \frac{2}{3}$

$\cos \Theta = \frac{2}{3}$



$\boxed{\tan\Theta = \frac{\sqrt{5}}{2}}$

(b) Find the exact value of $\sinh(\ln 3)$

$$\sinh(\ln 3) = \frac{e^{\ln 3} - e^{-\ln 3}}{2} = \frac{3 - \frac{1}{3}}{2}$$

$$= \frac{8}{6} = \boxed{\frac{4}{3}}$$

(c) Solve for x: $2 \log x - \log(x+1) = \log 4 - \log 3$

$$\frac{x^2}{x+1} = \frac{4}{3} \qquad 3x^2 = 4x + 4$$

$\boxed{x = 2}$

$3x^2 - 4x - 4 = 0$

$(3x+2)(x-2) = 0 \quad x = 2, \cancel{-\frac{2}{3}}$

(d) Solve for x: $e^{x^2} = \frac{e^{4x}}{e^3}$

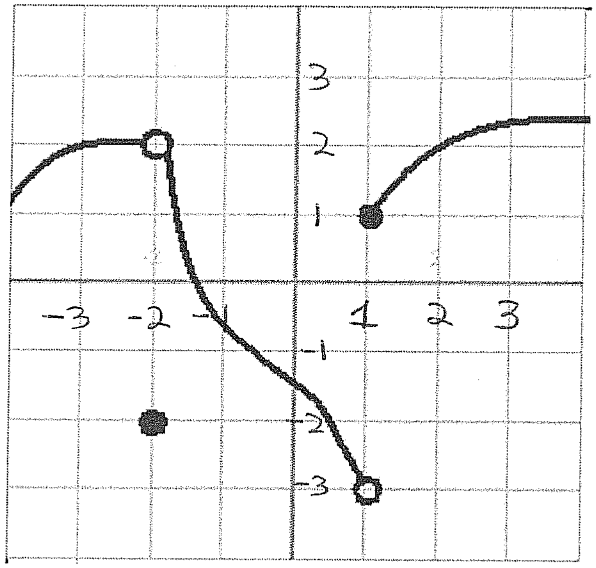
$x^2 = 4x - 3$

$x^2 - 4x + 3 = 0$

$\boxed{x = 1, 3}$

$(x-1)(x-3) = 0$

3. (10 points) Below is the graph of $f(x)$.



Find:

(a) $\lim_{x \to 1^+} f(x)$ = 1

(b) $\lim_{x \to 1^-} f(x)$ = -3

(c) $\lim_{x \to -2^+} f(x)$ = 2

(d) $\lim_{x \to -2^-} f(x)$ = 2

(e) $\lim_{x \to -2} f(x)$ = 2

4. (15 points)

(a) $\lim_{x \to 6} f(x) = 3$ and $\lim_{x \to 6} g(x) = 4$, find $\lim_{x \to 6} \frac{f(x)g(x) - 2}{3g(x) + 2}$.

$$= \frac{3 \cdot 4 - 2}{12 + 2} = \frac{10}{14} = \frac{5}{7}$$

(b) $\lim_{x \to -1} (3x^2 + 2x + 1)^5$

$$= 2^5 = 32$$

(c) $\lim_{x \to 16} \frac{\sqrt{x} - 4}{x - 16} \cdot \frac{\sqrt{x} + 4}{\sqrt{x} + 4}$

$$= \lim_{x \to 16} \frac{\cancel{x - 16}}{\cancel{x - 16}} \frac{1}{\sqrt{x} + 4} = \frac{1}{8}$$

5. (15 points)

(a) Find the value of a so that $f(x)$ is continuous if

$$f(x) = \begin{cases} x^2 + a & x \leq 0 \\ \\ \dfrac{\sin 2x}{x} & x > 0 \end{cases}$$

$$\lim_{x \to 0^-} f(x) = a$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{2 \sin 2x}{2x} = 2 \qquad a = 2$$

(b) $\displaystyle \lim_{x \to \pi/4} \frac{\sin x - \cos x}{1 - \tan x}$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{1 - \frac{\sin x}{\cos x}} \quad \frac{\cos x}{\cos x}$$

$$= \lim_{x \to \frac{\pi}{4}} \frac{\sin x - \cos x}{\cos x - \sin x} \cdot \cos x = -\frac{\sqrt{2}}{2}$$

(c) $\displaystyle \lim_{x \to \infty} \left(\ln(2x + 5) - \ln(\sqrt{x^2 + 7}) \right)$

$$= \lim_{x \to \infty} \ln\left(\frac{2x+5}{\sqrt{x^2+7}} \right) = \ln\left(\lim_{x \to \infty} \frac{2x+5}{\sqrt{x^2+7}} \right)$$

$$= \ln 2$$

6. (10 points)

(a) Given $f(x) = 2x + 3$ and $\lim_{x \to 1} f(x) = 5$, find a value of δ so that $|f(x) - 5| < 0.01$ whenever $0 < |x - 1| < \delta$

$|2x + 3 - 5| < 0.01$

$|2x - 2| < 0.01$

$2|x - 1| < 0.01$

$|x - 1| < \frac{0.01}{2} = .005$

$\boxed{\delta = 0.005}$

(b) Given $f(x) = 2x+3$ and $\lim_{x \to 1} f(x) = 5$, find a fomula for δ in terms of ϵ so that $|f(x) - 5| < \epsilon$ whenever $0 < |x - 1| < \delta$

$|2x + 3 - 5| < \varepsilon$

$|2x - 2| < \varepsilon$

$2|x - 1| < \varepsilon$

$|x - 1| < \frac{\varepsilon}{2}$

$\boxed{\delta = \frac{\varepsilon}{2}}$

7. (10 points) Indicate whether the following statements are true or false by circling the appropriate letter. A statement which is sometimes true and sometimes false should be marked false.

a) If $|f(x)|$ is continuous at $x = c$ then $f(x)$ is continuous at $x = c$. T F

b) If $\lim_{x \to c} f(x) = L$, then there is a δ such that if $0 < |x - c| < \delta$ then $|f(x) - L| < 0.001$ T F

c) If $f(x) < g(x)$ for all $x \neq c$, then $\lim_{x \to c} f(x) < \lim_{x \to c} g(x)$ T F

d) If $f(x) = g(x)$ for $x \neq c$ and $f(c) \neq g(c)$ then either $f(x)$ or $g(x)$ is not continuous at c. T F

e) If $f(1) = -2$ and $f(4) = 5$, then there is a c with $1 < c < 4$ where $f(c) = 0$ T F