Math 121 Test 1

September 15, 2015

EF:

1	
2	
3	
4	
5	
6	
7	
Total	

Name KEY

Directions:

1. No books, notes or Security Guards gone wild. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.
2. You should show your work and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.
3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.
4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.
5. This test has 7 problems.

1. (15 Points)

(a) Find the domain of $f(x) = \dfrac{\sqrt{2-x}}{x^2 - x}$

$x \neq 0, 1$ $\qquad$ $2 - x \geq 0$

$2 \geq x$

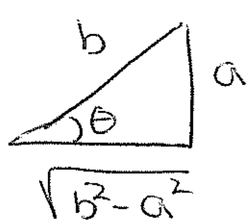
$(-\infty, 0) \cup (0, 1) \cup (1, 2]$

(b) Write $|x - 5| < 3$ in the form $a < x < b$.

$-3 < x - 5 < 3$

$2 < x < 8$

(c) Find $\tan\theta$ if $\sin\theta = \frac{a}{b}$



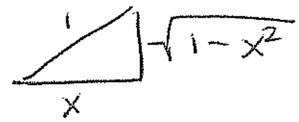
$\tan\theta = \dfrac{a}{\sqrt{b^2 - a^2}}$

2. (20 points)

(a) Which of the following is equal to $\cot(\arccos x)$?

a) $\dfrac{1}{\sqrt{1+x^2}}$ b) $\dfrac{\sqrt{1-x^2}}{x}$ $\boxed{\text{c) } \dfrac{x}{\sqrt{1-x^2}}}$ d) $\dfrac{1}{\sqrt{1-x^2}}$

$\Theta = \text{ARC COS } x$ $\cos\Theta = x$



$\cot\Theta = \dfrac{x}{\sqrt{1-x^2}}$

(b) Find $f^{-1}(x)$ for $f(x) = \frac{x+4}{7x-3}$ $x = \dfrac{y+4}{7y-3}$

$7xy - 3x = y + 4$

$7xy - y = 3x + 4$

$y(7x-1) = 3x+4$

$f^{-1}(x) = \dfrac{3x+4}{7x-1}$

(c) Solve for x: $\ln x^4 - \ln x^2 = 2$

$\ln x^2 = 2$

$x^2 = e^2$ $x = \pm e$

(d) Let $f(x) = \sqrt{x}$ and $g(x) = 1 - x$. Find:

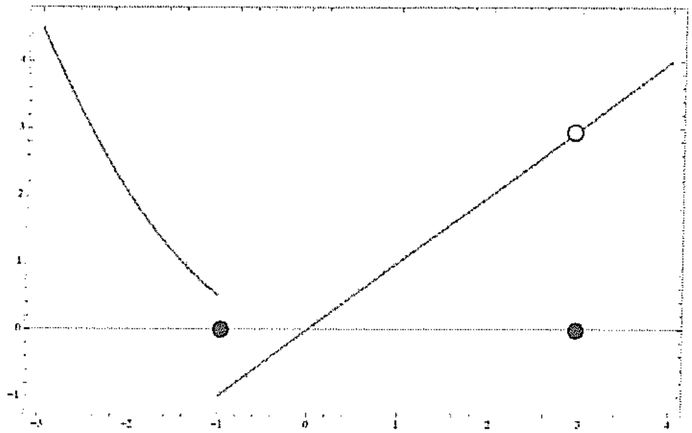
i. $f \circ g$

$= f(g(x)) = \sqrt{1-x}$

ii. $g \circ f$

$g(f(x)) = 1 - \sqrt{x}$

3. (10 points) Below is the graph of $f(x)$.



Find:

(a) $\lim_{x \to -1^+} f(x) = -1$

(b) $\lim_{x \to -1^-} f(x) = \frac{1}{2}$

(c) $\lim_{x \to -1} f(x) =$ DNE

(d) $f(-1) = 0$

(e) $\lim_{x \to 3} f(x) = 3$

4. (20 points)

(a) $\lim_{x\to 1} \frac{x^3-1}{5x-1}$ $= \frac{0}{4} = 0$

(b) $\lim_{x\to 2} \frac{2^{2x}+2^x-20}{2^x-4}$ $= \lim_{x \to 2} \frac{\cancel{(2^x-4)}(2^x+5)}{\cancel{2^x-4}} = 9$

(c) $\lim_{x\to 4} \left(\frac{1}{\sqrt{x}-2} - \frac{4}{x-4}\right)$

$$= \lim_{x\to 4} \frac{\sqrt{x}+2}{x-4} - \frac{4}{x-4} = \lim_{x\to 4} \frac{\cancel{\sqrt{x}-2}}{(\cancel{\sqrt{x}-2})(\sqrt{x}+2)}$$

$$= \frac{1}{4}$$

(d) $\lim_{x\to 5} \frac{x-5}{\sqrt{x+4}-3}$

$$= \lim_{x\to 5} \frac{x-5}{(\sqrt{x+4}-3)} \frac{\sqrt{x+4}+3}{(\sqrt{x+4}+3)} = \lim_{x\to 5} \frac{\cancel{(x-5)}(\sqrt{x+4}+3)}{\cancel{x-5}}$$

$$= 6$$

5. (15 points)

(a) Find the value of c so that $f(x)$ is continuous if

$$f(x) = \begin{cases} x^2 - c & x < 5 \\ 4x + 2c & x \geq 5 \end{cases}$$

$f(5) = 20 + 2c$

$\lim_{x \to 5^-} x^2 - c = 25 - c$

$\lim_{x \to 5^+} \cancel{20+2c} = 20 + 2c$

$25 - c = 20 + 2c$

$5 = 3c \qquad c = \frac{5}{3}$

(b) $\displaystyle\lim_{x \to 0} \frac{(\sin 5x)(\sin 4x)}{(\sin 3x)(\sin 2x)}$

$= \lim_{x \to 0} \frac{\sin 5x}{5x} \frac{3x}{\sin 3x} \frac{\sin(4x)}{4x} \frac{2x}{\sin 2x} \frac{5\cancel{x}}{3\cancel{x}} \frac{4\cancel{x}}{2\cancel{x}}$

$= \frac{20}{6} = \frac{10}{3}$

(c) $\displaystyle\lim_{x \to \infty} (\ln(3x+1) - \ln(2x+1))$

$= \lim_{x \to \infty} \ln\left(\frac{3x+1}{2x+1}\right)$

$\cancel{\ln} \lim_{x \to \infty} \frac{3x+1}{2x+1} = \ln\left(\frac{3}{2}\right)$

6. (10 points) Show that $\cos x = x$ has a solution in the interval $[0, 1]$. (Hint: Show that $f(x) = x - \cos x$ has a zero in $[0, 1]$).

$f(x)$ IS CONT.

$f(0) = -1$

$f(1) = 1 - \cos 1 > 0$

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$f(c) = 0$ FOR $0 \leq c \leq 1$

7. (10 points) Indicate whether the following statements are true or false by circling the appropriate letter. A statement which is sometimes true and sometimes false should be marked false.

a) If $\lim_{x \to c} f(x) = L$, then $f(c) = L$. T F

b) If $\lim_{x \to c} f(x) = L$, then $\dfrac{\lim_{x \to c^-} f(x)}{\lim_{x \to c^+} f(x)} = 1$ T F

c) If $f(x)$ has a discontinuity as $x = c$, then $\lim_{x \to c^-} f(x)$ does not exist. T F

d) If $h(x) < f(x) < g(x)$ and $\lim_{x \to c} h(x) = \lim_{x \to c} g(x) = L$, then $\lim_{x \to c} f(x) = L$. T F

e) If $f(x)$ is a polynomial and $f(1) = -2$ and $f(4) = 5$, then there is a c with $1 < c < 4$ where $f(c) = 0$ T F