Math 121 Test 1

September 20, 2022

EF:

Name KEY

1	
2	
3	
4	
5	
Total	

Directions:

1. No books, notes or teaching with shoes on. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.
2. You should show your work and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.
3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.
4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.
5. This test has 5 problems.

1. (20 Points)

(a) If you write $|3x-5|<2$ in the form $a<x<b$, what are a and b?

$-2 < 3x-5 < 2$

$3 < 3x < 7$

$1 < x < \frac{7}{3}$

$\boxed{a = 1 \quad b = \frac{7}{3}}$

(b) Find the domain of $f(x) = \dfrac{x+x^{-1}}{(x+2)(x-3)}$

$\boxed{x \neq 0, -2, 3}$

(c) Find y so that $(3, y)$ is on the line with slope $m = 2$ and goes through $(1, 4)$.

$y - 4 = 2(x-1)$

$y - 4 = 2(3-1)$

$y - 4 = 4$ $\boxed{y = 8}$

(d) If $f(x) = 2^x$ and $g(x) = x^2$, find both $(f \circ g)(x)$ and $(g \circ f)(x)$.

$(f \circ g)(x) = f(g(x)) = f(x^2) = \boxed{2^{x^2}}$

$(g \circ f)(x) = g(f(x)) = g(2^x) = (2^x)^2$

$= \boxed{2^{2x}}$

2. (20 points)

(a) Solve for x, $e^3 \, e^{x^2} = e^{4x}$

$$x^2 + 3 = 4x$$

$$x^2 - 4x + 3 = 0$$

$$(x-1)(x-3) = 0$$

$$\boxed{x = 1, 3}$$

(b) Find the inverse of the function $f(x) = \frac{x+3}{5x-3}$

$$y = \frac{x+3}{5x-3} \qquad x = \frac{y+3}{5y-3} \qquad (5y-3)(x) = y+3$$

$$5yx - 3x = y + 3 \qquad 5yx - y = 3x + 3$$

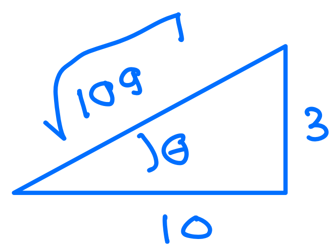
$$y(5x-1) = 3x + 3$$

$$\boxed{f^{-1}(x) = \frac{3x+3}{5x-1}}$$

(c) If $\cot\theta = \frac{10}{3}$ and $0 \le \theta \le \frac{\pi}{2}$ find

i. $\sin\theta$

$$\boxed{\sin\theta = \frac{3}{\sqrt{109}}}$$



ii. $\sec\theta$

$$\boxed{\sec\theta = \frac{\sqrt{109}}{10}}$$

3. (20 points)

(a) Find $\lim_{x\to 1} \frac{5-x^2}{4x+7}$ $= \boxed{\frac{4}{11}}$

(b) Find $\lim_{x\to 1}(x-1)\sin\left(\frac{\pi}{x^2-1}\right)$.

$$\boxed{\lim_{x\to 1}(x-1)\sin\left(\frac{\pi}{x^2-1}\right)=0}$$

$$-1 \le \sin\left(\frac{\pi}{x^2-1}\right) \le 1$$

$$\underbrace{-(x-1)}_{\to 0} \le (x-1)\sin\left(\frac{\pi}{x^2-1}\right) \le \underbrace{(x-1)}_{\to 0}$$

(c) Find $\lim_{x\to a} \frac{x^3-ax^2-x+a}{x-a}$

$$= \lim_{x\to a} \frac{x^2(x-a)-(x-a)}{(x-a)}$$

$$= \lim_{x\to a} \frac{\cancel{(x-a)}(x^2-1)}{\cancel{(x-a)}} = \boxed{a^2-1}$$

(d) Find $\lim_{x\to 0} \frac{\sin 3x}{2x^2+5x}$

$$= \lim_{x\to 0} \frac{\sin 3x}{3x} \cdot \left(\frac{1\cdot 3}{2x+5}\right) = \boxed{\frac{3}{5}}$$

4. (20 points)

(a) $\lim_{x\to 1}\left(\frac{1}{x-1}-\frac{2}{x^2-1}\right)$

$$= \lim_{x\to 1} \frac{x+1-2}{(x-1)(x+1)} = \lim_{x\to 1} \frac{\cancel{x-1}\ 1}{\cancel{(x-1)}(x+1)}$$

$$= \boxed{\frac{1}{2}}$$

(b) $\lim_{h\to 0}\frac{\sqrt{a+2h}-\sqrt{a}}{h}$ $\quad \frac{\sqrt{a+2h}-\sqrt{a}}{\sqrt{a+2h}-\sqrt{a}}$

$$= \lim_{h\to 0} \frac{\cancel{a}+2\cancel{h}-\cancel{a}}{\cancel{h}(\sqrt{a+2h}+\sqrt{a})} = \frac{\cancel{2}}{\cancel{2}\sqrt{a}} = \boxed{\frac{1}{\sqrt{a}} \text{ OR } \frac{\sqrt{a}}{a}}$$

(c) $\lim_{x\to 7^-}\frac{|x-7|}{x-7}$ $= \lim_{x\to 7} \frac{-\cancel{(x-7)}}{\cancel{x-7}} = \boxed{-1}$

(d) $\lim_{x\to\infty}\left(x-\sqrt{x^2+7x}\right)$ $\left(\frac{x+\sqrt{x^2+7x}}{x+\sqrt{x^2+7x}}\right)$

$$= \lim_{x\to 0} \frac{\cancel{x^2}-(\cancel{x^2}+7x)}{x+\sqrt{x^2+7x}} = \boxed{-\frac{7}{2}}$$

5. (20 points) Indicate whether the following statements are true or false by circling the appropriate letter. A statement which is sometimes true and sometimes false should be marked false.

a) If $f(x) = \dfrac{|x+1|}{x+1}$ then $f(x) = 0$ for $-2 < x < 1$ **T** **F**

b) If $f(x)$ is continuous on $[0, 2]$ and $f(0) = 2$ and $f(2) = 5$ Then $f(x) \neq 0$ on $(0, 2)$. **T** **F**

c) If $f(c)$ is not defined, then $f(c)$ is not continuous at $x = c$ **T** **F**

d) If $\lim_{x \to c} f(x)$ does not exist, then $f(x)$ is not continuous at $x = c$. **T** **F**

e) If $f(x)$ and $g(x)$ are continuous everywhere, then $h(x) = \dfrac{f(x)}{g(x)}$ is continuous everywhere. **T** **F**

f) If $f(x) > g(x)$ for all $x \neq c$ then $\lim_{x \to c} f(x) > \lim_{x \to c} g(x)$ **T** **F**

g) The function $f(x) = \dfrac{x+1}{x^2 - x - 2}$ is continuous at $x = -1$. **T** **F**

h) If $3x - 2 \leq f(x) \leq x^2$ for $0 \leq x \leq 3$ then $\lim_{x \to 2} f(x) = 4$ **T** **F**

i) If $\lim_{x \to 5} f(x) = 2$, then there is a number $\delta > 0$ such that if $|x - 5| < \delta$, then $|f(x) - 2| < 0.001$ **T** **F**

j) $1 + 1 = 2$ **T** **F**