

Math 122

Test 1 - Review 1

I. Substitution

1. $\int \frac{1}{x(\ln x)^2} dx$

2. $\int \frac{x^2 - 2x^4}{x^4} dx$

3. $\int x^4 \sqrt[3]{3 - 5x^5} dx$

4. $\int x^2 \sqrt{x - 1} dx$

II. Parts

1. $\int x^2 \cos x dx$

2. $\int (\ln x)^3 dx$

3. $\int e^x \sin x dx$

4. $\int \ln x^3 dx$

III. Trig. Integrals

1. $\int \sin^2 x \cos^3 x dx$

2. $\int \sec^5 x \tan^3 x dx$

3. $\int \cos^2 3x dx$

4. $\int \frac{\sin^2 x}{\cos x} dx$

IV. Trig. Substitution

1. $\int \frac{1}{\sqrt{9 + x^2}} dx$

2. $\int \frac{1}{(25 - x^2)^{3/2}} dx$

3. $\int \frac{1}{\sqrt{x^2 - 4}} dx$

4. $\int \frac{x^3}{\sqrt{x^2 - 1}} dx$

V. Hyperbolic Functions

1. $\int \sinh^2 x \cosh x dx$

2. $\int \tanh(3x) \operatorname{sech}(3x) dx$

VI. Partial Fractions

1. $\int \frac{3x+4}{(x^2+4)(3-x)} dx$

2. $\int \frac{2}{(x+2)^2(2-x)} dx$

3. $\int \frac{x+1}{x^2-5x+6} dx$

4. $\int \frac{8+9(\ln x)^2}{x(\ln x)^3+x \ln x} dx$

VII. Improper Integrals

1. $\int_1^4 \frac{1}{(x-2)^4} dx$

2. $\int_0^9 \frac{1}{\sqrt{9-x}} dx$

3. $\int_0^2 \frac{dx}{x^2-4x+3}$

4. $\int_{-\infty}^{\infty} \frac{1}{1+4x^2} dx$

5. $\int_0^{\infty} \frac{x}{x^4+1} dx$

6. $\int_0^{\infty} \frac{1}{\sqrt{x}(x+1)} dx$

VIII. Numerical Integration

Use Simpson's method with $n = 10$ to evaluate:

1. $\int_{-1}^1 e^{-x^2} dx$

2. $\int_0^1 \cos(x^2) dx$

IX. Miscellaneous

1. $\int \frac{x^2}{x^2+2} dx$

2. $\int \frac{\tan^2 x}{\sec^3 x} dx$

3. $\int \frac{dx}{e^x+5+4e^{-x}}$

4. $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}} dx$

5. $\int \frac{x^2+2}{(x-2)(x+1)^2} dx$

6. $\int \frac{1}{1-e^{-x}} dx$

7. $\int \frac{1}{1-\sin x} dx$

8. $\int \frac{x}{(x-3)^{2/3}} dx$

Answers

Section I.

1. $-\frac{1}{\ln x} + C$
2. $-\frac{1}{x} - 2x + C$
3. $-\frac{3}{100}(3 - 5x^5)^{4/3} + C$
4. $\frac{2}{7}(x-1)^{7/2} + \frac{4}{5}(x-1)^{5/2} + \frac{2}{3}(x-1)^{3/2} + C$

Section II.

1. $x^2 \sin x + 2x \cos x - 2 \sin x + C$
2. $x(\ln x)^3 - 3x(\ln x)^2 + 6x(\ln x) - 6x + C$
3. $\frac{1}{2}e^x (\sin x - \cos x) + C$
4. $3x \ln x - 3x + C$

Section III.

1. $\frac{1}{3} \sin^3 x - \frac{1}{5} \sin^5 x + C$
2. $\frac{1}{7} \sec^7 x - \frac{1}{5} \sec^5 x + C$
3. $\frac{1}{2} \left[x + \frac{1}{6} \sin 6x \right] + C$
4. $\ln |\sec x + \tan x| - \sin x + C$

Section IV.

1. $\ln |\sqrt{x^2 + 9} + x| + C$
2. $\frac{x}{25\sqrt{25 - x^2}} + C$
3. $\ln |x + \sqrt{x^2 - 4}| + C$
4. $\sqrt{x^2 - 1} + \frac{1}{3}(x^2 - 1)^{3/2} + C$

Section V.

1. $\frac{1}{3} \sinh^3 x + C$

2. $-\frac{1}{3} \operatorname{sech}(3x) + C$

Section VI.

1. $\ln \left| \frac{\sqrt{x^2 + 4}}{3 - x} \right| + C$
2. $\frac{1}{8} \ln \left| \frac{x+2}{2-x} \right| - \frac{1}{2(x+2)} + C$
3. $4 \ln |x - 3| - 3 \ln |x - 2| + C$
4. $8 \ln[\ln x] + \frac{1}{2} \ln[(\ln x)^2 + 1] + C$

Section VII.

1. D.N.E.
2. 6
3. D.N.E.
4. $\frac{\pi}{2}$
5. $\frac{\pi}{4}$
6. π

Section VIII.

1. 1.49367
2. 0.904524

Section IX.

1. $x - \sqrt{2} \arctan \frac{x}{\sqrt{2}} + C$
2. $\frac{\sin^3 x}{3} + C$
3. $\frac{1}{3} \ln |e^x + 1| - \frac{1}{3} \ln |e^x + 4| + C$
4. $\arcsin(\tan x) + C$
5. $\frac{2}{3} \ln |x - 2| + \frac{1}{3} \ln |x + 1| + \frac{1}{x+1} + C$
6. $\ln |e^x - 1| + C$
7. $\tan x + \sec x + C$
8. $\frac{3}{4}(x - 3)^{4/3} + 9(x - 3)^{1/3} + C$