Quiz Book Number

Math 121 Test 3

November 19, 2024

EF:

1	
2	
3	
4	
5	
Total	

Name KEY

Directions:

1. No books, notes or unicycle TikToc videos. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.

2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.

3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.

4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

5. This test has 5 problems.

1. (20 points)

(a) Use a linear approximation to estimate $\sqrt[5]{33}$.

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$f(x) = \sqrt[5]{x}$ $a = 32$ $f(32) = 2$

$f'(x) = \frac{1}{5}x^{-\frac{4}{5}}$ $f'(a) = \frac{1}{5(16)} = \frac{1}{80}$

$L(x) = f(a) + f'(a)(x-a) = 2 + \frac{1}{80}(x-32)$

$\sqrt[5]{33} = f(33) \approx L(33) = 2 + \frac{1}{80}(33-32) = \frac{161}{80} = 2.0125$

(b) Find the maximum and minimum values for $f(x) = x^2 - 8\ln x$ on the interval $[1, 4]$

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$f'(x) = 2x - \frac{8}{x}$ $2x - \frac{8}{x} = 0$ $x^2 = 4$ $x = \pm 2$

x	$f(x)$	
1	1	
2	$4 - 8\ln 2 = -1.545$	MIN
4	$16 - 8\ln 4 = 4.91$	MAX

(c) If using Newton's Method to find where

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$$4 + 8x^2 - x^4 = 0$$

Circle which of the following that would **NOT** be a good place to start Newton's Method (there may be more than one)?

i) $x = -3$ ii) $x = -2$ iii) $x = -1$ iv) $x = 0$

v) $x = 1$ vi) $x = 2$ vii) $x = 3$

$f(x) = 4 + 8x^2 - x^4$

$f'(x) = 16x - 4x^3 = 0$

$4x(4 - x^2) = 0$

$x = 0, \pm 2$

2. (20 points) For $f(x) = \dfrac{x}{x^2 - 4}$

Hint: $f'(x) = -\dfrac{x^2+4}{(x^2-4)^2}$ and $f''(x) = \dfrac{2x(x^2+12)}{(x^2-4)^3}$

find:

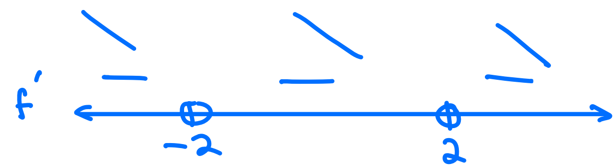
2

(a) Domain: $x \neq \pm 2$

(b) Range: ALL y

(c) x-intercepts: $y = 0$

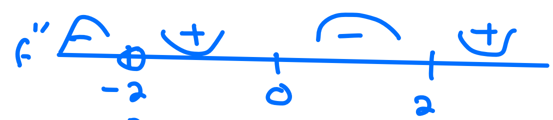
(d) y-intercepts: $x = 0$



2 (e) Where y is increasing: NO WHERE

2 (f) Where y is decreasing: $(-\infty, -2) \cup (-2, 2) \cup (2, \infty)$

2 (g) Critical points: NONE



2 (h) Where y is concave up: $(-2, 0) \cup (2, \infty)$

2 (i) Where y is concave down: $(-\infty, -2) \cup (0, 2)$

2 (j) Inflection points: $x = 0$ ONLY

2 (k) Vertical asymptotes: $x = \pm 2$

2 (l) Horizontal asymptotes: $y = 0$

(m) Sketch the graph of y

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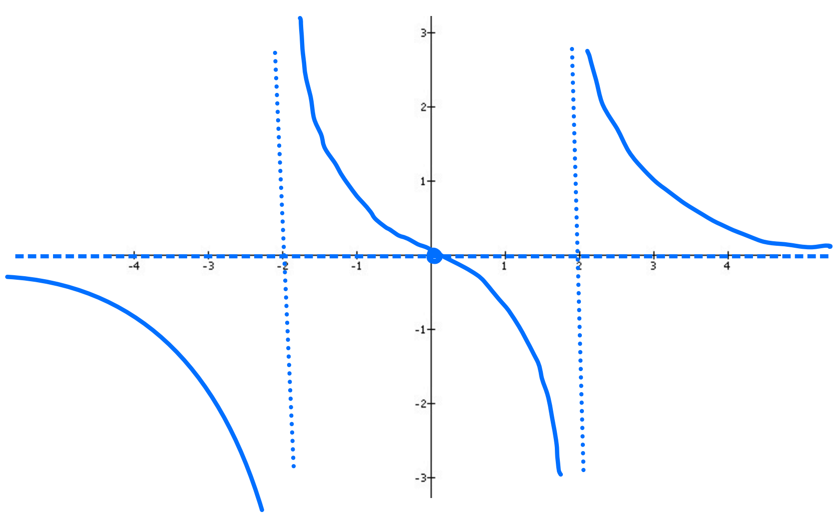

3. (20 points)

(a) Chez Bútlěř, a very fancy restaurant is trying to decide how much to charge per seat for reservations on Thanksgiving. Experience has shown that at a cost of \$200 per seat, all 80 seats in the restaurant will be booked. On average, there will be one fewer customer for each \$20 increase in price above \$200. What price should be charge to maximize revenue?

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$$\begin{array}{r} 80 \\ 20 \\ \hline 1600 \end{array}$$

x = # OF \$20 INC.

$R = \text{PRICE} \cdot \#$

$\text{PRICE} = 200 + 20x \qquad \# = 80 - x$

$R = (200 + 20x)(80 - x) = 16{,}000 + 1400x - 20x^2$

$\frac{dR}{dx} = 1400 - 40x = 0 \qquad x = 35$

$\text{PRICE} = 200 + (35)(20) = \$900$

$\# = 80 - 35 = 45$

x	R
0	16,000
45	40,500
80	0

(b) Compute $\lim_{x \to 0} \frac{e^x + e^{-x} - 2}{1 - \cos x} = \text{"}\frac{0}{0}\text{"}$

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$\lim_{x \to 0} \frac{e^x - e^{-x}}{\sin x} = \text{"}\frac{0}{0}\text{"} \qquad \lim_{x \to 0} \frac{e^x + e^{-x}}{\cos x} = 2$

(c) Find $\lim_{x \to \pi/4} (1 - \tan x) \sec 2x$

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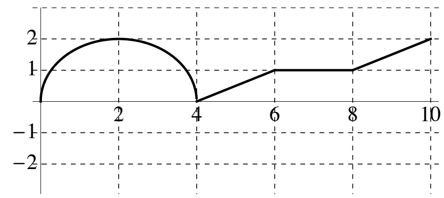
$= \lim_{x \to \frac{\pi}{4}} \frac{1 - \tan x}{\cos 2x} = \text{"}\frac{0}{0}\text{"} \qquad \lim_{x \to 0} \frac{-\sec^2 x}{-2 \sin 2x} = 1$

4. (20 points)

(a) The graph of a piecewise defined function $f(x)$ consisting of a semicircle and 3 straight lines, is shown below.



i. Use the graph to calculate the value of R_5, the right endpoint approximation to $\int_0^{10} f(x)\ dx$ using 5 equal subintervals

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$$= 2(2) + 2(0) + 2(1) + 2(1) + 2(2)$$

$$= 12$$

ii. Compute $\int_0^{10} f(x)\ dx$

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$$= \frac{\pi(2)^2}{2} + \frac{1}{2}(2)(1) + 2(1) + \frac{1+2}{2}(2)$$

$$= 2\pi + 6$$

(b) Konrad was writing a question for this test. The answer was

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$$\int f(x)\ dx = e^x \arctan x + C$$

but he can't remember what $f(x)$ was supposed to be. Please help Konrad and find $f(x)$

$$f(x) = F'(x)$$

$$F(x) = e^x \text{ ARCTAN } x$$

$$F'(x) = \frac{e^x}{1+x^2} + e^x \text{ ARCTAN } x$$

5. (20 points)

(a) Compute: $\int (4-3x^2)(4x+1)\,dx$

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$$= \int (16x - 12x^3 + 4 - 3x^2)\,dx$$

$$= 8x^2 - 3x^4 + 4x - x^3 + C$$

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(b) Compute $\int x^2\sqrt{x-1}\,dx$ $\quad u = x-1 \quad du = dx$

$x = u+1$

$$= \int (u+1)^2\sqrt{u}\,du = \int (u^2+2u+1)\sqrt{u}\,du$$

$$= \int u^{\frac{5}{2}} + 2u^{\frac{3}{2}} + u^{\frac{1}{2}}\,du = \frac{2u^{\frac{7}{2}}}{7} + \frac{2\cdot 2u^{\frac{5}{2}}}{5} + \frac{2u^{\frac{3}{2}}}{3}$$

$$= \frac{2}{7}(x-1)^{\frac{7}{2}} + \frac{4}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + C$$

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(c) Compute: $\int_0^{\sqrt{\pi}} x\sin(x^2)\,dx$

$u = x^2 \quad du = 2x\,dx \quad dx = \frac{du}{2x}$

$$= \int \cancel{x}\sin(u)\,\frac{du}{2\cancel{x}} = -\frac{1}{2}\cos u = -\frac{1}{2}\cos(x^2)\Big|_0^{\sqrt{\pi}}$$

$$= -\frac{1}{2}(-1-1) = 1$$

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(d) Compute $\int \frac{\sec^2 x}{\sqrt{1-\tan^2 x}}\,dx$ $\quad u = \tan x$

$du = \sec^2 x\,dx$

$$= \int \frac{\cancel{\sec^2 x}}{\sqrt{1-u^2}}\,\frac{du}{\cancel{\sec^2 x}} = \arcsin u$$

$$= \arcsin(\tan x) + C$$