Math 121
Test 3 - Review 1

Use differentials to approximate the following. Compare your answer to that of a calculator.

1. $\sqrt{99.4}$ **2.** $\sqrt[3]{28}$ **3.** $\sqrt[4]{624}$

4. Consider the graph of the equation $f(x) = \frac{2x^2-1}{x^2}$

a. Find $f'(x)$ and $f''(x)$.

b. Find the domain of $f(x)$.

c. Find any vertical or horizontal asymptotes.

d. Find all x and y intercepts.

e. Find where f is increasing, where f is decreasing, and any critical points.

f. Find where f is concave up, where f is concave down, and any inflection points.

g. Sketch the graph of $f(x)$.

5. Sketch a graph of a function with the following properties (if possible), or say it is not possible:

a. a continuous function on the interval $(1, 3)$ with a minimum but no maximum.

b. a continuous function on the interval $[1, 3]$ with both a maximum and minimum.

c. a continuous function on the interval $[1, 3]$ with a minimum but no maximum.

6. Use the following information to sketch the graph. Label all critical points with "c.p." and all inflection points with "i.p.".

Domain is $(-\infty, 2) \cup (2, \infty)$, $f(-1) = 3, f(-2) = 2$

x-intercepts: $(1, 0)$, $(3, 0)$, y-intercepts: $(0, 2)$,

Vertical asymptotes: $x = 2$, $\lim_{x\to\infty} f(x) = 1$, $\lim_{x\to-\infty} f(x) = 0$

$f'(x) < 0$ for x in $(-1, 2)$, $f'(-1) = 0$ $f'(x) > 0$ for x in $(-\infty, -1) \cup (2, \infty)$

$f''(x) < 0$ for x in $(-2, 2) \cup (2, \infty)$, $f''(-2) = 0$ $f''(x) > 0$ for x in $(-\infty, -2)$

7. Find the maximum and minimum values for $f(x) = 5x^{2/3} - x^{5/3}$ on the interval $[-1, 4]$

8. Find the maximum and minimum of $f(x) = 6x^{4/3} - 3x^{1/3}$ on the interval $[-1, 1]$.

9. Find the maximum and minimum values for $f(x) = x + \frac{4}{x}$ on the interval $[1, 5]$

10. A closed cylindrical can is to hold 1000 cubic cm. of liquid. What should be the height and radius of the can to minimize the total surface area.

11. A vertically challenged resident of Puerto Rico wants to fence in an area of 5400 square feet in a rectangular field and then divide it in half with a fence down the middle parallel to one side. What is the shortest length of fence that can be used?

12. A piece of sheet metal is rectangular, 5 ft wide and 8 ft long. Congruent squares are to be cut from its four corners. The resulting piece of metal is to be folded and welded to form an open topped box. How should this be done to get a box of largest possible volume?

13. Prof. Meyer and Dean Mason have agreed to a charity sumo wrestling match. From experience we know 200 people will show up if the ticket price is \$10.00. For each 10 cent increase, the number attending will decrease by 10 (and for each 10 cent decrease, the number attending will increase by 10). What price should be set to maximize the amount raised for charity?

Compute the following limits:

14. $\lim_{x\to 0} \dfrac{\sin x - x}{x^3}$

15. $\lim_{x\to \infty} e^{-x} \ln x$

16. $\lim_{x\to 0} \dfrac{e^{x^2} - 1}{2x^2}$

17. $\lim_{x\to \infty} \dfrac{x^2}{\ln x}$

18. $\lim_{x\to 0} \dfrac{1 - \cos x}{x - \sin x}$

19. $\lim_{x\to 0} \dfrac{e^x - e^{-x}}{\sin x}$

20. $\lim_{x\to \infty} \left(1 + \dfrac{2}{x}\right)^x$

21. $\lim_{x\to \infty} [\ln x - \ln(x+1)]$

22. $\lim_{x\to 0} (1+2x)^{-3/x}$

23. True or False: Newton's method works to find where $2x^3 + 3x^2 - 12x + 5 = 0$ starting at $x_0 = 1$

24. If you were to use Newton's method to find where $x^2 - 80 = 0$ and started at $x_1 = 9$ what is $x_2 =$?

25. Use Rolle's Theorem to show that the polynomial $f(x) = x^3 + x + 1$ has only one root.

26. Given that

$$\int_0^1 f(x)\,dx = \frac{1}{2} \qquad \int_1^2 f(x)\,dx = \frac{1}{4} \qquad \int_0^3 f(x)\,dx = -1 \qquad \int_0^1 g(x)\,dx = 2$$

Find

a. $\int_0^2 f(x)\,dx$

b. $\int_1^3 f(x)\,dx$

c. $\int_2^3 5f(x)\,dx$

d. $\int_1^0 g(x)\,dx$

Use areas where appropriate to find:

27. $\int_{-1}^{1} (1 + \sqrt{1-x^2})\,dx$

28. $\int_0^1 x\sqrt{1-x^4}\,dx$

Compute the following indefinite integrals:

29. $\int x\sqrt{x}\ dx$

30. $\int \frac{(\ln x)^2}{x}\ dx$

31. $\int \frac{(1+x)^2}{\sqrt{x}}\ dx$

32. $\int x\sqrt{x+1}\ dx$

33. $\int \tan^3 x \sec^2 x\, dx$

34. $\int x + x^2 + \frac{1}{x^2}\ dx$

35. $\int (1+x^3)^2\, dx$

36. $\int \frac{\cos x}{\sin^3 x} dx$

37. $\int x^2 + \frac{1}{\sqrt{x}}\, dx$

38. $\int \sqrt{2+x}\, dx$

39. $\int \cos x \tan x\, dx$

40. $\int \frac{1}{4+x^2}\, dx$

41. $\int 3^{-x}\, dx$

42. $\int \frac{1}{\sqrt{1-16x^2}}\, dx$

Compute the following definite integrals:

43. $\int_1^2 \frac{x^2}{x^3+1}\ dx$

44. $\int_0^{\pi/4} e^{\sin 2x}\ \cos 2x\ dx$

45. $\int_0^1 x\sqrt{9x^2+16}\ dx$

46. $\int_0^{10} \sqrt{10x-x^2}\, dx$

47. $\int_{-\pi/8}^{\pi/8} \frac{1}{\cos^2\theta}\ d\theta$

48. $\int_0^{\pi/2} \sin 2x\ \cos 2x\ dx$

49. $\int_1^3 \frac{3x}{\sqrt{x+2}}\ dx$

50. $\int_0^3 (x^3-6x)\ dx$

51. $\int_{-1}^0 6t^2(t^3+1)^{19}\, dt$

52. Compute $\int_{-2}^{2} f(x)\, dx$ where $f(x) = \begin{cases} 3, & x \le 0 \\ x+3, & x > 0 \end{cases}$

53. Find $F'(x)$ and $F''(x)$ for

$$F(x) = \int_3^{x^2} \sin(t^2+1)\ dt$$

54. Find $f'(x)$ and $f''(x)$ for

$$f(x) = \int_2^x \sqrt{t^3 + 2t + 5}\, dt$$

55. Find the derivatives of the following functions:

a. $F(x) = \int_0^x \sin(t^2)\, dt$

b. $F(x) = \int_x^{10} \cos(t^2)\, dt$

c. $F(x) = \int_{\sqrt{x}}^{10} \tan(t^2)\, dt$

d. $F(x) = \int_5^{10} \sec(t^2)\, dt$

e. $y = \int_1^{\ln x} \frac{1}{t^2}\, dt$

f. $y = \int_{\ln 2}^{\ln 3} \frac{\cos t}{\sqrt{t}}\, dt$

For $f(x) = x^2 + 2$ from $x = 0$ to $x = 4$

56. Estimate the area under by dividing the interval $[0, 4]$ into $n = 4$ subintervals of equal length and then compute

$$\sum_{i=1}^{4} f(x_k^*)\Delta x$$

with x_k^* as the right endpoint of each subinterval.

57. Compute the area under $y = x^2 + 2$ from $x = 0$ to $x = 4$ using the Fundamental Theorem of Calculus.

Find the limit by interpreting the limit of Riemann sums on the interval $[0, 1]$ is divided into n subintervals of equal length

58. $\lim_{n\to\infty} \frac{\sqrt{1} + \sqrt{2} + \sqrt{3} + \ldots + \sqrt{n}}{n^{3/2}}$

59. $\lim_{n\to\infty} \frac{1^4 + 2^4 + 3^4 + \ldots + n^4}{n^5}$

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Answers

1. $\sqrt{99.4} \approx 9.97$ (Calculator: 9.9699)

2. $\sqrt[3]{28} \approx 3.03704$ (Calculator: 3.03658)

3. $\sqrt[4]{624} \approx 4.998$ (Calculator: 4.997998)

4. **a.** $f'(x) = \frac{2}{x^3}$ and $f''(x) = -\frac{6}{x^4}$.

b. $x \neq 0$

c. Vertical asymptotes: $x = 0$, Horizontal asymptotes: $y = 2$

d. x-int.: $x = \pm\sqrt{\frac{1}{2}}$, y-int.: None.

e. Increasing: $(0, \infty)$, Decreasing: $(-\infty, 0)$ Critical points: None

f. Concave up: no where, Concave down: $(-\infty, 0) \cup (0, \infty)$ Inflection points: None

7. Maximum of 6 at $x = -1$ and minimum of 0 at $x = 0$.

8. Maximum of 9 at $x = -1$, minimum of $-\frac{9}{8}$ at $x = \frac{1}{8}$

9. Maximum of $\frac{29}{5}$ at $x = 5$ and minimum of 4 at $x = 2$.

10. $R = \sqrt[3]{\frac{1000}{2\pi}} \approx 5.42, h = \frac{1000}{\sqrt[3]{250000\pi}} \approx 10.83$

11. $F ==$ 360 feet

12. To get the maximum volume, you should cut squares of 1 ft by 1 ft out of each corner.

13. \$6.00 per person

14. $-\frac{1}{6}$

15. 0

16. $\frac{1}{2}$

17. ∞

18. D.N.E.

19. 2

20. e^2

21. 0

22. e^{-6}

23. False

24. 8.944

25. Since $f(x) = x^3 + x + 1$ is a cubic polynomial, it must have at least one root. If it had two roots, then between them, by Rolle's theorem, the derivative would have to equal zero. Since the derivative is $f'(x) = 3x^2 + 1$ never equals zero, there can only be one root.

26. **a.** $\frac{3}{4}$

b. $-\frac{3}{2}$

c. $-\frac{35}{4}$

d. -2

27. $2 + \frac{\pi}{2}$

28. $\frac{\pi}{8}$

29. $\frac{2}{5}x^{5/2} + C$

30. $\frac{(\ln x)^3}{3} + C$

31. $2\sqrt{x} + \frac{4}{3}x^{3/2} + \frac{2}{5}x^{5/2} + C$

32. $\frac{2}{5}(x+1)^{5/2} + \frac{2}{3}(x+1)^{3/2} + C$

33. $\frac{\tan^4 x}{4} + C$

34. $\frac{x^2}{2} + \frac{x^3}{3} - \frac{1}{x} + C$

35. $x + \frac{1}{2}x^4 + \frac{x^7}{7} + C$

36. $-\frac{1}{2\sin^2 x} + C$

37. $\frac{x^3}{3} + 2\sqrt{x} + C$

38. $\frac{2(2+x)^{3/2}}{3} + C$

39. $-\cos x + C$

40. $\frac{1}{2}\arctan\frac{x}{2} + C$

41. $-\frac{3^{-x}}{\ln 3} + C$

42. $\frac{1}{4}\arcsin 4x + C$

43. $\frac{1}{3}[\ln 9 - \ln 2]$

44. $\frac{e-1}{2}$

45. $\frac{61}{27}$

46. $\frac{25\pi}{2}$

47. 0.828

48. 0

49. $6\sqrt{3} - 2\sqrt{5}$

50. $-\frac{27}{4}$

51. $\frac{1}{10}$

52. 14

53. $F'(x) = 2x\sin(x^4+1) \qquad F''(x) = 2\sin(x^4+1) + 8x^4\cos(x^4+1)$

54. $f'(x) = \sqrt{x^3+2x+5} \qquad f''(x) = \frac{1}{2}\frac{3x^2+2}{\sqrt{x^3+2x+5}}$

55. **a.** $F'(x) = \sin(x^2)$

b. $F'(x) = -\cos(x^2)$

c. $F'(x) = -\frac{1}{2}x^{-1/2}\left(\tan(x)\right)$

d. $F'(x) = 0$

e. $y' = \frac{1}{(\ln x)^2}\frac{1}{x}$

f. $y' = 0$

56. $A \approx 1(3+6+11+18) = 38$

57. $A = \int_0^4 x^2 + 2\ dx = \frac{x^3}{3} + 2x\Big|_0^4 = \frac{64}{3} + 8 = \frac{88}{3}$

58. $\frac{2}{3}$

59. $\frac{1}{5}$