Math 121 Test 1

EF:

September 14, 2021

Name KEY

1	
2	
3	
4	
5	
Total	

Directions:

1. No books, notes or writing exams during Brown's games. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.

2. You should show your work and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.

3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.

4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

5. This test has 5 problems.

1. (20 Points)

(a) If you write $|2x - 7| < 9$ in the form $a < x < b$, what are a and b?

$-9 < 2x - 7 < 9$

$-2 < 2x < 16$ $\quad a = -1$

$-1 < x < 8$ $\quad b = 8$

(b) If $f(x)$ has a domain of $[2, 6]$ and range of $[1, 5]$, what is the domain and range of $f(x + 2) + 3$?

DOMAIN $[0,4]$ SHIFT DOWN 2

RANGE $[4,8]$ SHIFT UP 2

(c) What is the equation of the line perpendicular to $2x + 3y = 7$ through $(1, 1)$?

$3y = -2x + 7 \quad y = -\frac{2}{3}x + \frac{7}{3}$

$m = -\frac{2}{3} \quad m_{\perp} = \frac{3}{2}$

$y - 1 = \frac{3}{2}(x - 1)$

(d) If $f(x) = x^2 + x$ and $g(x) = 3x + 5$, what is $(f \circ g \circ f)(1)$?

$f(g(f(1))) = f(g(2)) =$

$f(11) = 121 + 11 = 132$

2. (20 points)

(a) Solve for x, $\ln(x) + \ln(x-1) = 0$

$$\ln[(x)(x-1)] = 0 \qquad x(x-1) = e^0 = 1$$

$$x^2 - x - 1 = 0 \qquad x = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1 \pm \sqrt{5}}{2}$$

SINCE x MUST BE POSITIVE $\quad x = \frac{1+\sqrt{5}}{2}$

(b) Find the inverse of the function $f(x) = \frac{x}{2x+3}$

$$y = \frac{x}{2x+3}$$

$$x = \frac{y}{2y+3}$$

$$x(2y+3) = y$$

$$2xy + 3x = y$$

$$2xy - y = -3x$$

$$y(2x-1) = -3x$$

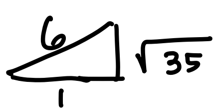
$$y = \frac{-3x}{2x-1}$$

$$f^{-1}(x) = \frac{-3x}{2x-1}$$

(c) What is $\tan(\arccos \frac{1}{6})$?

$$\theta = \text{ARCCOS}\left(\frac{1}{6}\right)$$

$$\text{COS}\,\theta = \frac{1}{6}$$



$$\text{TAN}\,\theta = \sqrt{35}$$

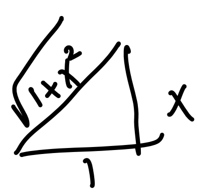
(d) What is $\sin(\arctan x)$?

(a) $\frac{1}{\sqrt{1-x^2}}$ (b) $\frac{1}{\sqrt{x^2-1}}$ (c) $-\frac{x}{\sqrt{x^2+1}}$ $\boxed{\text{(d) } \frac{x}{\sqrt{1+x^2}}}$

(e) None of these

$$\theta = \text{ARCTAN}\,x$$

$$\text{TAN}\,\theta = x$$



$$\text{SIN}\,\theta = \frac{x}{\sqrt{1+x^2}}$$

3. (20 points)

(a) Find $\lim_{x \to 5} \frac{\sqrt{x+4}+1}{\sqrt{x-1}-1}$ $= \frac{3+1}{2-1} = 4$

(b) Find $\lim_{x \to 0} (x^3+x)\cos\left(\frac{1}{x^3+x}\right)$. $= 0$ BY SQUEEZE THM

$$-1 \leq \cos\left(\frac{1}{x^3+x}\right) \leq 1$$

$$\underbrace{-(x^3+x)}_{\to 0} \leq (x^3+x)\cos\left(\frac{1}{x^3+x}\right) \leq \underbrace{(x^3+x)}_{\to 0}$$

(c) Find $\lim_{x \to 4} \frac{1-\frac{16}{x^2}}{1-\frac{4}{x}}$ $\frac{x^2}{x^2}$

$$= \lim_{x \to 4} \frac{x^2-16}{x^2-4x} = \lim_{x \to 4} \frac{\cancel{(x-4)}(x+4)}{x\cancel{(x-4)}}$$

$$= \frac{8}{4} = 2$$

(d) Find $\lim_{x \to 0} \frac{\sin^2 8x}{\tan^2 7x}$

$$= \lim_{x \to 0} \frac{\sin 8x}{8x} \frac{\sin 8x}{8x} \frac{7x}{\sin 7x} \frac{7x}{\sin 7x} \frac{\cos^2 7x}{1} \frac{64\cancel{x^2}}{49\cancel{x^2}}$$

$$= \frac{64}{49}$$

4. (20 points)

(a) $\lim_{x\to 1} \frac{\sqrt{x}-1}{3-\sqrt{8+x}} \cdot \frac{\sqrt{x}+1}{\sqrt{x}+1} \; \frac{3+\sqrt{8+x}}{3+\sqrt{8+x}}$

$$= \lim_{x\to 1} \frac{(x-1)(3+\sqrt{8+x})}{(9-(8+x))(\sqrt{x}+1)} = \lim_{x\to 1} \frac{\overset{-1}{\cancel{(x-1)}}(3+\sqrt{8+x})}{\cancel{(1-x)}(\sqrt{x}+1)}$$

$$= -\frac{6}{2} = -3$$

(b) $\lim_{x\to 1} \frac{3^{2x}-7(3^x)+12}{3^x-3}$

$$= \lim_{x\to 1} \frac{\cancel{(3^x-3)}(3^x-4)}{\cancel{(3^x-3)}} = 3-4 = -1$$

(c) $\lim_{x\to\infty} \frac{3x^3-3x+23}{5x^4-5x^2+625x+15}$

$= 0$

POWER ON TOP 3

POWER ON BOTTOM 4

(d) $\lim_{x\to -\infty} \frac{3x+2}{\sqrt{x^2+x+1}} = -3$

POWER TOP AND BOTTOM 1

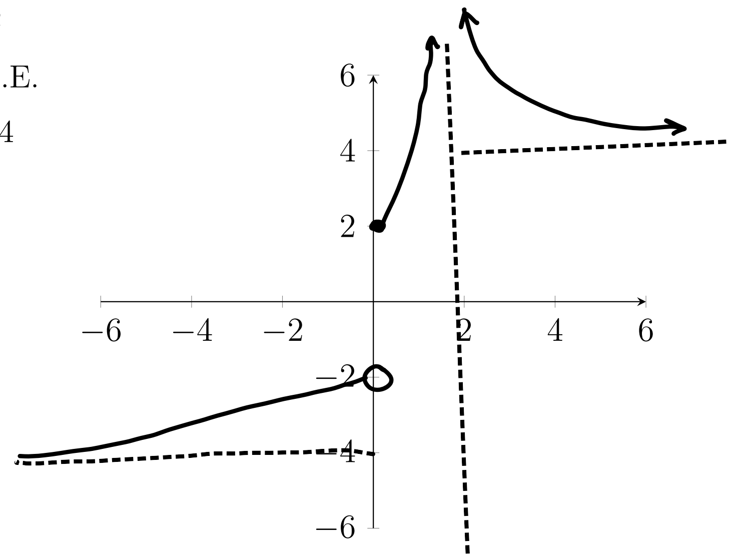
TOP NEGATIVE

BOTTOM POSITIVE

5. (20 points)

(a) On the axis below, sketch a graph of a function that meets all the following criteria:

- $\lim_{x\to 0^+} f(x) = 2$
- $\lim_{x\to 0^-} f(x) = -2$
- $\lim_{x\to 2} f(x) = \text{D.N.E.}$
- $\lim_{x\to -\infty} f(x) = -4$
- $\lim_{x\to +\infty} f(x) = 4$



(b) Indicate whether the following statements are true or false by circling the appropriate letter. A statement which is sometimes true and sometimes false should be marked false.

a) If $f(x)$ is continuous on $[-1,1]$ and $f(-1) = 4$ and $f(1) = 3$ then there is a c where $f(c) = \pi$ **T** **F**

b) If $\lim_{x\to 0} f(x) = 0$ and $\lim_{x\to 0} g(x) = 0$ then $\dfrac{\lim_{x\to 0} f(x)}{\lim_{x\to 0} g(x)}$ cannot exist. **T** **F**

c) If $\lim_{x\to 1} f(x) = 3$, then there is a δ such that if $0 < |x-1| < \delta$ then $|f(x) - 3| < 2$ **T** **F**

d) If $\lim_{x\to\infty} \dfrac{\sin x}{x} = 1$ **T** **F**

e) If $f^{-1}(x)$ exists, and $f(2) = 2$ then $f^{-1}(2) = \frac{1}{2}$ **T** **F**