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Math 122 Test 3

April 14, 2026

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Total	

Name KEY

Directions:

1. No books, notes or drawing comical pictures of your math instructor. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.

2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.

3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.

4. Numerical experiments do not count as justification. For example, computing the first few terms of a series is not enough to show the terms decrease. Computing the first few partial sums of a series is not enough to show the series converges or diverges. Plugging in numbers is not enough to justify the computation of a limit.

5. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

1. (15 points)

(a) Find the limit of the following sequence:

$$\left\{\frac{\ln(n+2)}{\ln(1+4n)}\right\}_{n=1}^{\infty}$$

$$\lim_{N\to\infty} \frac{\ln(N+2)}{\ln(1+4N)} = \lim_{N\to\infty} \frac{\frac{1}{N+2}}{\frac{4}{1+4N}}$$

$$= \lim_{N\to\infty} \frac{1+4N}{4(N+2)} = 1$$

(b) Find the sum of the following series:

i. $\displaystyle\sum_{n=0}^{\infty}\left(\frac{6^n}{7^n}+\frac{(-1)^n}{4^n}\right)$ $= \displaystyle\sum_{N=0}^{\infty}\left(\frac{6}{7}\right)^N + \sum_{N=0}^{\infty}\left(\frac{-1}{4}\right)^N$

$$= \frac{1}{1-\frac{6}{7}} + \frac{1}{1-\left(\frac{-1}{4}\right)} =$$

$$7 + \frac{4}{5} = \frac{39}{5}$$

ii. $\displaystyle\sum_{n=1}^{\infty}\left(\frac{1}{n+2}-\frac{1}{n+4}\right)$

$$= \left(\frac{1}{3}-\cancel{\frac{1}{5}}\right)+\left(\frac{1}{4}-\cancel{\frac{1}{6}}\right)+\left(\cancel{\frac{1}{5}}-\cancel{\frac{1}{7}}\right)+\left(\cancel{\frac{1}{6}}-\cancel{\frac{1}{8}}\right)\cdots$$

$$= \frac{1}{3}+\frac{1}{4} = \frac{7}{12}$$

2. (15 points) For each of the following series, determine if it converges or diverges. For each test you use, you must name the test, perform the test, and state the conclusion you reached from that test.

(a) $\sum_{n=1}^{\infty} \frac{(n+1)^2}{(n^2+2)^2}$ L.C.T $\leq \frac{1}{N^2}$ $P = 2 > 1$

$\lim_{N\to\infty} \frac{(N+1)^2}{(N^2+2)^2} \cdot \frac{N^2}{1} = 1$

SINCE $\sum_{N=1}^{\infty} \frac{1}{N^2}$ CONV. $\Rightarrow \sum_{N=1}^{\infty} \frac{(N+1)^2}{(N^2+2)^2}$ CONV.

(b) $\sum_{n=2}^{\infty} \frac{5+3\sin n}{\sqrt{n-1}}$ COMP TEST

$\frac{5+3\sin N}{\sqrt{N-1}} > \frac{2}{\sqrt{N}}$ $\leq \frac{2}{\sqrt{N}} = 2\sum \frac{1}{\sqrt{N}}$ DIV $P = \frac{1}{2} < 1$

$\Rightarrow \sum_{N=2}^{\infty} \frac{5+3\sin N}{\sqrt{N-1}}$ DIV.

(c) $\sum_{n=1}^{\infty} \frac{(n!)^2}{(2n+1)!}$ RATIO TEST

$\lim_{N\to\infty} \frac{((N+1)!)^2}{(2N+3)!} \cdot \frac{(2N+1)!}{(N!)^2}$

$= \lim_{N\to\infty} \frac{(N+1)^2}{(2N+2)(2N+3)} = \frac{1}{4} < 1$

$\Rightarrow \sum_{N=1}^{\infty} \frac{(N!)^2}{(2N+1)!}$ CONV.

3. (15 points) Determine if the following series converge absolutely, converge conditionally, or diverge. For each test you use, you must name the test, perform the test, and state the conclusion you reached from that test.

(a) $\sum_{n=2}^{\infty} \frac{(-1)^n n^3}{3^n}$ RATIO TEST

$$\lim_{N\to\infty} \frac{(N+1)^3}{3^{N+1}} \cdot \frac{3^N}{N^3} = \frac{1}{3} < 1$$

$\Rightarrow$ CONV. ABS.

(b) $\sum_{n=2}^{\infty} \frac{(-1)^n}{1-e^{-n}}$ N-TH TERM TEST

$$\lim_{N\to\infty} \frac{1}{1-e^{-N}} = 1 \neq 0$$

DIV.

(c) $\sum_{n=2}^{\infty} \frac{(-1)^n}{n \ln n}$

INT TEST $\int_2^{\infty} \frac{1}{x \ln x}\, dx = \ln(\ln x)\Big|_2^{\infty} = \text{DIV}$

$\Rightarrow$ A.S.T

A.S.T. $\lim_{N\to\infty} \frac{1}{N \ln N} = 0$ ✓

$\frac{1}{(N+1)\ln(N+1)} < \frac{1}{N \ln N}$ ✓

CONV. COND.

4. (15 points) Consider the power series:

$$\sum_{n=1}^{\infty} \frac{(x-3)^n}{2^n \sqrt{n}}$$

(a) Where is the power series centered?

$c = 3$

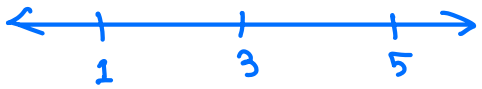
(b) Find the radius of convergence.

$$\lim_{N\to\infty} \left| \frac{(x-3)^{N+1}}{2^{N+1}\sqrt{N+1}} \cdot \frac{2^N \sqrt{N}}{(x-3)^N} \right| = \left| \frac{x-3}{2} \right| < 1$$

$|x-3| < 2$

$R = 2$

(c) Find the interval of convergence. [Hint: Check the endpoints.]



$x = 5$ $\quad \sum_{N=1}^{\infty} \frac{(5-3)^N}{2^N \sqrt{N}} = \sum_{N=1}^{\infty} \frac{1}{\sqrt{N}}$ DIV $p = \frac{1}{2} < 1$

$x = 1$ $\quad \sum_{N=1}^{\infty} \frac{(1-3)^N}{2^N \sqrt{N}} = \sum_{N=1}^{\infty} \frac{(-1)^N}{\sqrt{N}}$ AST

$\lim_{N\to\infty} \frac{1}{\sqrt{N}} = 0$ ✓ $\quad \frac{1}{\sqrt{N+1}} < \frac{1}{\sqrt{N}}$ ✓

CONV. COND

$[1, 5)$

5. (20 points) Given the Maclaurin series for $\tan x$ is

$$\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} + \dots$$

(a) Find the first 4 non-zero terms for Maclaurin Series for $f(x) = x^2 \tan(2x)$.

$$\tan(2x) = 2x + \frac{8x^3}{3} + \frac{64x^5}{15} + \frac{2176x^7}{315} \dots$$

$$x^2 \tan(2x) = 2x^3 + \frac{8x^5}{3} + \frac{64x^7}{15} + \frac{2176x^9}{315} \dots$$

(b) Find the first 4 non-zero terms for Maclaurin for $g(x) = \sec^2 x$.

$$\frac{d}{dx}(\tan x) = \sec^2 x$$

$$\sec^2 x = 1 + x^2 + \frac{10x^4}{15} + \frac{119x^6}{315} \dots$$

(c) Find $T_5(x)$ the fifth degree Maclaurin polynomial for $\tan x$.

$$T_5(x) = x + \frac{x^3}{3} + \frac{2x^5}{15}$$

(d) Find $\lim_{x \to 0} \dfrac{\tan x - x}{x^3}$

$$\lim_{x \to 0} \frac{x + \frac{x^3}{3} + \frac{2x^5}{15} + \frac{17x^7}{315} - x}{x^3}$$

$$= \frac{1}{3}$$

6. (20 points)

(a) If $c(t) = (e^t, \frac{2}{3}e^{3t/2})$. find the length of the curve for $\ln 3 \le t \le \ln 8$.

$x = e^t \quad \frac{dx}{dt} = e^t \quad \left(\frac{dx}{dt}\right)^2 = e^{2t}$

$y = \frac{2}{3}e^{\frac{3t}{2}} \quad \frac{dy}{dt} = e^{\frac{3t}{2}} \quad \left(\frac{dy}{dt}\right)^2 = e^{3t}$

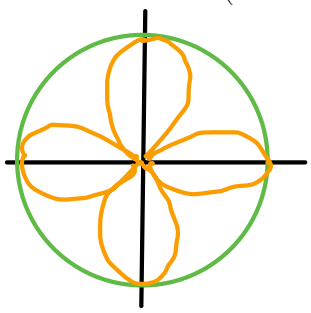
$$S = \int_{\ln 3}^{\ln 8} \sqrt{e^{2t} + e^{3t}}\, dt = \int_{\ln 3}^{\ln 8} e^t \sqrt{1 + e^t}\, dt$$

$u = 1 + e^t \quad du = e^t\, dt \quad dt = \frac{du}{e^t}$

$$= \int e^t \sqrt{u}\, \frac{du}{e^t} = \frac{2}{3}u^{\frac{3}{2}} =$$

$$\frac{2}{3}(1+e^t)^{\frac{3}{2}}\Big|_{\ln 3}^{\ln 8} = \frac{2}{3}(27 - 8) = \frac{38}{3}$$

(b) Find the area of the region inside $r = 2$ and outside $r = 2\cos(2\theta)$. (Hint: Sketch the graph)



$A = A_{CIRCLE} - A_{CLOVER}$

$$= \pi(2)^2 - \int_0^{2\pi} \frac{(2\cos(2\theta))^2}{2}\, d\theta$$

$$= 4\pi - 2\int_0^{2\pi} \cos^2(2\theta)\, d\theta = 4\pi - 2\int_0^{2\pi} \frac{1 - \cos 4\theta}{2}\, d\theta$$

$$= 4\pi - \left(\theta - \frac{\sin 4\theta}{2}\right)\Big|_0^{2\pi} = 4\pi - 2\pi$$

$$= 2\pi$$