SI: | | EF: | |

Math 122 Test 2

October 13, 2026

Name KEY

1	
2	
3	
4	
5	
Total	

Directions:

1. No books, notes or your dog getting sprayed by a skunk. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.

2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.

3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.

4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

1. (10 points) Let $p(x) = \dfrac{C}{\sqrt{1-x^2}}$ for $0 \le x \le 1$.

   (a) Find C so $p(x)$ is a probability density function.

$$\int_0^1 \frac{C}{\sqrt{1-x^2}}\,dx = C\int_0^1 \frac{1}{\sqrt{1-x^2}}\,dx = C\arcsin x\Big|_0^1$$

$$= C\left(\frac{\pi}{2}\right) = 1 \qquad \boxed{C = \frac{2}{\pi}}$$

   (b) Find the mean μ.

$$\mu = \frac{2}{\pi}\int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx \qquad u = 1-x^2 \qquad du = -2x\,dx \qquad dx = \frac{du}{-2x}$$

$$= \frac{2}{\pi}\int \frac{x}{\sqrt{u}}\,\frac{du}{-2x} = \frac{-1}{\pi}\,2\sqrt{u} = -\frac{2}{\pi}\left(1-x^2\right)^{\frac{1}{2}}\Big|_0^1$$

$$\boxed{\mu = \frac{2}{\pi}}$$

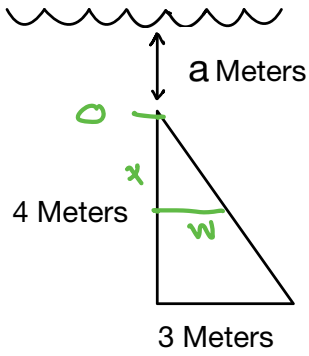
2. (10 points) Find the arc length of the curve $y = 2(x-1)^{3/2}$ for $1 \le x \le 8$.

$$y' = 3(x-1)^{\frac{1}{2}} \qquad (y')^2 = 9(x-1) \qquad 1+(y')^2 = 9x-8$$

$$S = \int_1^8 \sqrt{9x-8}\,dx = \frac{2(9x-8)^{\frac{3}{2}}}{3\cdot 9}\Big|_1^8$$

$$= \frac{2}{27}\left[512-1\right] = \boxed{\frac{1022}{27}}$$

3. (10 points) Kelsey has a triangular plate submerged in a water of weight density (ρg) = 9810 N/ m^3. She knows the total force on the plate is 274,680 N. How far is the top of the triangle under the surface of the water (what is the value of a)?



$h(x) = a + x$

$\frac{w}{x} = \frac{3}{4}$

$w = \frac{3x}{4}$

$F = \int_a^b (\rho g) h(x)\, w(x)\, dx$

$F = \int_0^4 (9810)(a+x)\left(\frac{3x}{4}\right) dx$

$= \frac{3(9810)}{4}\int_0^4 ax + x^2\, dx = \frac{3(9810)}{4}\left[\frac{ax^2}{2} + \frac{x^3}{3}\right]_0^4$

$\frac{3(9810)}{4}\left[8a + \frac{64}{3}\right] = 274,680$

$8a + \frac{64}{3} = \frac{112}{3}$ $\quad 8a = 16$ $\quad \boxed{a = 2}$

4. (10 points) Find $\bar{y}$, the y coordinate of the centroid, for the region lying between $y = e^x$ and the x-axis for $0 \le x \le 1$.

$M = \int_0^1 e^x\, dx = e^x\Big|_0^1 = e - 1$

$M_x = \int_0^1 \frac{(e^x)^2}{2}\, dx = \frac{1}{2}\int_0^1 e^{2x}\, dx = \frac{e^{2x}}{4}\Big|_0^1 = \frac{1}{4}[e^2 - 1]$

$\bar{y} = \frac{\frac{1}{4}[e^2-1]}{e-1} = \frac{\frac{1}{4}(e-1)(e+1)}{e-1} = \boxed{\frac{1}{4}(e+1)}$

5. (10 points) Solve $y' = xe^{x^2-y}$ $\quad y(0) = 0$

$$\frac{dy}{dx} = \frac{x e^{x^2}}{e^y} \qquad \int e^y \, dy = \int x e^{x^2} \, dx$$

$$e^y = \frac{e^{x^2}}{2} + C$$

$$y = \ln\left(\frac{e^{x^2}}{2} + C\right)$$

$$\boxed{y = \ln\left(\frac{e^{x^2}}{2} + \frac{1}{2}\right)}$$

$$e^0 = e^{\ln\left(\frac{1}{2} + C\right)}$$

$$1 = \frac{1}{2} + C \qquad C = \frac{1}{2}$$

6. (10 points) Charles leaves his ice-tea out in the sun. The temperature of the tea is initially 5°C. After 5 minutes, the tea has warmed to 10°C where the air temperature is 30°C.

(a) Determine the temperature as a function of time.

$$\frac{dy}{dt} = -k(y - T_0) \qquad y = T_0 + C e^{-kt}$$

$$y = 30 + C e^{-kt} \qquad 5 = 30 + C e^{-k(0)} \qquad C = -25$$

$$y = 30 - 25 e^{-kt} \qquad 10 = 30 - 25 e^{-k(5)}$$

$$k = 0.0446$$

$$\boxed{y = 30 - 25 e^{-.0446\,t}}$$

(b) What is the temperature after 10 minutes?

$$y(10) = 30 - 25 e^{(-.0446)(10)} = \boxed{14^\circ C}$$

(c) Determine when the tea will reach a temperature of 20°C.

$$20 = 30 - 25 e^{-.0446t}$$

$$\boxed{t = 20.5 \text{ min.}}$$

7. (10 points) Use Euler's method with $h = \frac{1}{2}$ to approximate $y(2)$ if

$$\frac{dy}{dx} = 1 + xy \qquad y(1) = 3$$

x	y	$F(x,y)$	$hF(x,y)$	$y + hF(x,y)$
1	3	4	2	5
1.5	5	8.5	4.25	9.25
2	9.25			

$y(2) \approx 9.25$

8. (10 points) Suppose Caleb is in charge of a Camel Camp.The population of Camels follows the logistic equation

$$\frac{dy}{dt} = .02y - 0.0001y^2 \qquad y(0) = 50$$

where t is measured in weeks. $\frac{dy}{dt} = .02y\left(1 - \frac{y}{200}\right)$

(a) How many Camels are at the Camp after 50 weeks?

$A = 200$

$y_0 = 50$

$K = .02$

$B = \frac{50}{50-200} = \frac{50}{-150} = -\frac{1}{3}$

$$y = \frac{200}{1 + 3e^{-.02t}}$$

$$y(50) = \frac{200}{1 + 3e^{(-.02)(50)}}$$

$= 95$ CAMELS

(b) After a long time ($t \to \infty$), how many Camels are at the Camp?

$$\lim_{t \to \infty} \frac{200}{1 + 3e^{-.02t}} = 200 \text{ CAMELS}$$

9. (10 points) Solve $y' + 2y - e^{2x} = 0 \qquad y(0) = 3$

$y' + 2y = e^{2x}$

$P(x) = 2 \quad Q(x) = e^{2x} \quad \alpha(x) = e^{\int 2dx} = e^{2x}$

$y = \frac{1}{e^{2x}}\left[\int e^{2x} e^{2x}\, dx + C\right] = e^{-2x}\left[\frac{1}{4}e^{4x} + C\right]$

$3 = \frac{1}{4} + C$

$C = \frac{11}{4}$

$$\boxed{y = e^{-2x}\left[\frac{1}{4}e^{4x} + \frac{11}{4}\right]}$$

10. (10 points) Anthony mixes a tank that contains 30 L of a salt solution. Water flows into the tank at the rate of 3L/min that contains 2 grams/L of salt, and the well-stirred mixture flows out at a rate of 3 L/min. There is initially 20 grams of salt in the tank.

(a) Find a differential equation with initial condition for the amount of salt in the tank.

$\frac{dy}{dt} = 2 \cdot 3 - 3\frac{y}{30} \qquad y(0) = 20$

$$\boxed{\frac{dy}{dt} + \frac{1}{10}y = 6 \qquad y(0) = 20}$$

(b) Solve the differential equation from part (a).

$P(t) = \frac{1}{10} \quad Q(t) = 6 \quad \alpha(t) = e^{\int \frac{1}{10}dt} = e^{\frac{1}{10}t}$

$y = \frac{1}{e^{\frac{1}{10}t}}\left[\int e^{\frac{1}{10}t} \cdot 6\, dt + C\right] = e^{-\frac{1}{10}t}\left[60e^{\frac{1}{10}t} + C\right]$

$y = 60 + Ce^{-\frac{1}{10}t} \qquad 20 = 60 + Ce^{-\frac{1}{10}\cdot 0} \qquad C = -40$

$$\boxed{y = 60 - 40e^{-\frac{1}{10}t}}$$

(c) How much salt did is in the tank after 5 minutes?

$y(5) = 60 - 40e^{-\frac{1}{10}\cdot 5}$

$$\boxed{= 35.7 \text{ g}}$$