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# Math 122 Test 1

**September 15, 2026**

EF: [ ]

**Name** KEY

| | |
|---|---|
| 1 | |
| 2 | |
| 3 | |
| 4 | |
| 5 | |
| Total | |

Directions:

1. No books, notes or adding an extra second to the end of the game. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.

2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.

3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.

4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

1. (10 points) $\int \sqrt{\frac{x^6}{x^4+12}}\,dx$ $= \int \frac{x^3}{\sqrt{x^4+12}}\,dx$

$u = x^4+12$

$du = 4x^3\,dx$ $\qquad dx = \frac{du}{4x^3}$

$= \int \frac{x^3}{\sqrt{u}}\,\frac{du}{4x^3} = \frac{1}{4}\,2\sqrt{u} = \boxed{\frac{\sqrt{x^4+12}}{2} + C}$

2. (10 points) If $f(2) = 3$ and $\int_0^2 f(x)\,dx = 5$, find $\int_0^2 x\,f'(x)\,dx$

$\int_0^2 x\,f'(x)\,dx = x\,f(x)\Big|_0^2 - \int_0^2 f(x)\,dx$

$u = x \quad dv = f'(x)\,dx$

$du = dx \quad v = f(x)$

$= 2(3) - 0 - 5 = 6 - 5 \quad \boxed{= 1}$

3. (10 points) $\displaystyle\int \frac{\cos^3 x}{(1-\cos^2 x)^2}\,dx$ $= \displaystyle\int \frac{\cos^3 x}{\sin^4 x}\,dx$

$u = \sin x \quad du = \cos x\,dx \quad dx = \dfrac{du}{\cos x}$

$= \displaystyle\int \frac{\cos^2 x}{u^4}\,\frac{du}{\cos x} = \int \frac{1-u^2}{u^4}\,du$

$= \displaystyle\int u^{-4} - u^{-2}\,du = \frac{u^{-3}}{-3} + \frac{u^{-1}}{1}$

$$= \frac{(\sin x)^{-3}}{-3} + (\sin x)^{-1} + C$$

4. (10 points) $\displaystyle\int (\sec^5 x)(\tan^3 x)\,dx$

$u = \sec x \quad du = \sec x \tan x\,dx$

$dx = \dfrac{du}{\sec x \tan x}$

$= \displaystyle\int u^4 \tan^2 x\,\frac{du}{\sec x \tan x} = \int u^4 (u^2-1)\,du$

$= \displaystyle\int u^6 - u^4\,du = \frac{u^7}{7} - \frac{u^5}{5}$

$$= \frac{\sec^7 x}{7} - \frac{\sec^5 x}{5} + C$$

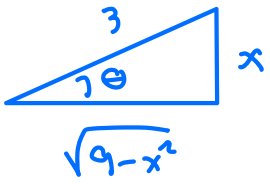
5. (10 points) $\int \frac{1}{x^2\sqrt{9-x^2}}\,dx$

$x = 3\sin\theta$

$dx = 3\cos\theta\, d\theta$

$$= \int \frac{3\cos\theta\, d\theta}{9\sin^2\theta\sqrt{9-9\sin^2\theta}} = \frac{1}{9}\int \csc^2\theta\, d\theta$$

$$= -\frac{1}{9}\cot\theta = \boxed{-\frac{1}{9}\frac{\sqrt{9-x^2}}{x} + C}$$




6. (10 points) $\int x^2 \sinh x\, dx$

$u = x^2 \qquad dv = \sinh x\, dx$

$du = 2x\, dx \qquad v = \cosh x$

$$= x^2\cosh x - \int 2x\cosh x\, dx$$

$u = 2x \qquad dv = \cosh x\, dx$

$du = 2\, dx \qquad v = \sinh x$

$$= x^2\cosh x - \left(2x\sinh x - \int 2\sinh x\, dx\right)$$

$$\boxed{= x^2\cosh x - 2x\sinh x + 2\cosh x + C}$$

7. (10 points) $\int \frac{x^2+x+4}{(x+1)(x-1)^2}\,dx$

$$\frac{x^2+x+4}{(x+1)(x-1)^2} = \frac{A}{x+1} + \frac{B}{x-1} + \frac{C}{(x-1)^2} = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

$$x^2+x+4 = A(x-1)^2 + B(x+1)(x-1) + C(x+1)$$

$x=1$ $\quad 6 = 2C \quad C=3$

$x=-1$ $\quad 4 = 4A \quad A=1$

$x=0$ $\quad 4 = 1 - B + 3 \quad B=0$

$$= \int \frac{1}{x+1} + \frac{3}{(x-1)^2}\,dx = \boxed{\ln|x+1| - \frac{3}{x-1} + C}$$

8. (10 points) $\int \frac{1}{x-3\sqrt{x}+2}\,dx$ (Hint: Let $u=\sqrt{x}$)

$u = \sqrt{x} \quad du = \frac{1}{2\sqrt{x}}\,dx \quad dx = 2u\,du$

$$= \int \frac{2u}{u^2-3u+2}\,du$$

$$\frac{2u}{(u-1)(u-2)} = \frac{A}{u-1} + \frac{B}{u-2} = \frac{A(u-2)+B(u-1)}{(u-1)(u-2)}$$

$u=2 \quad 4 = B$

$u=1 \quad 2 = -A \quad A=-2$

$$= \int \frac{-2}{u-1} + \frac{4}{u-2}\,du = -2\ln|u-1| + 4\ln|u-2|$$

$$\boxed{= -2\ln|\sqrt{x}-1| + 4\ln|\sqrt{x}-2| + C}$$

9. (10 points) $\displaystyle\int_e^{\infty} \frac{1}{x(\ln x)^2 + x}\, dx$

$u = \ln x$

$du = \frac{1}{x}\, dx$ $\quad dx = x\, du$

$= \displaystyle\int \frac{\cancel{x}\, du}{\cancel{x}(u^2+1)} = \text{ARCTAN}(u)$

$= \text{ARCTAN}(\ln x)\Big|_e^{\infty} = \frac{\pi}{2} - \frac{\pi}{4} = \boxed{\frac{\pi}{4}}$

10. (10 points) Use the chart below and Trapezoid method with $n = 6$ to approximate the integral

$$\int_2^8 f(x)\, dx$$

| $x$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 |
|---|---|---|---|---|---|---|---|
| $f(x)$ | 6 | 8 | 2 | $-4$ | 1 | $-2$ | 4 |
| | 1 | 2 | 2 | 2 | 2 | 2 | 1 |

$\Delta x = \frac{b-a}{n}$

$= \frac{8-2}{6}$

$= 1$

$6 + 16 + 4 - 8 + 2 - 4 + 4$

$= 20 \cdot \frac{\Delta x}{2}$

$= 20 \cdot \frac{1}{2}$ $\quad \boxed{= 10}$