Quiz Book Number

Math 121 Test 2

October 13, 2026

EF:

1	
2	
3	
4	
5	
Total	

Name KEY

Directions:

1. No books, notes or your dog getting sprayed by a skunk. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.

2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.

3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.

4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

5. This test has 5 problems.

1. (20 points) Find the derivatives of the following functions:

(a) $f(x) = 3\sqrt[7]{x^2} - \frac{2}{\sqrt{x}}$ $= 3x^{\frac{2}{7}} - 2x^{-\frac{1}{2}}$

$f'(x) = 3\left(\frac{2}{7}\right)x^{-\frac{5}{7}} - 2\left(-\frac{1}{2}\right)x^{-\frac{3}{2}}$

$= \frac{6}{7}x^{-\frac{5}{7}} + x^{-\frac{3}{2}}$

(b) $f(x) = (e^{3x})(\sin 2x)$

$f'(x) = e^{3x}(\cos 2x)(2) + 3e^{3x}\sin 2x$

(c) $f(x) = \frac{x^2+1}{x^2-3x-1}$

$f'(x) = \frac{(x^2-3x-1)(2x) - (x^2+1)(2x-3)}{(x^2-3x-1)^2}$

(d) $f(x) = [(3+x^2)^3+3]^4$

$f'(x) = 4\left[(3+x^2)^3+3\right]^3\left[3(3+x^2)^2(2x)\right]$

2. (20 points)

(a) Find $f'(1)$ if $f(x+h) - f(x) = h^2x^2 + 3hx + h\cos h$.

$$f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h} = \lim_{h\to 0} \frac{h^2x^2 + 3hx + h\cos h}{h}$$

$$f'(x) = 3x + 1 \qquad f'(1) = 3 + 1 = \boxed{4}$$

(b) If the equation of the tangent line to $f(x)$ at $x = 2$ is $y = -2x + 5$

i. Find $f(2) + f'(2)$

$$f(2) = -2(2) + 5 = 1$$
$$f'(2) = -2$$

$$f(2) + f'(2) = 1 - 2 = \boxed{-1}$$

ii. If $g(x) = (f(x))^3$, find $g'(2)$.

$$g'(x) = 3(f(x))^2 \cdot f'(x)$$

$$g'(2) = 3(1)^2(-2) = \boxed{-6}$$

(c) $f(x)$ and $g(x)$ have values shown below.

x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
3	-6	4	6	2
4	2	-1	7	-3

i. If $q(x) = \dfrac{f(x)}{g(x)}$ find $q'(3)$

$$q'(3) = \frac{g(3)f'(3) - f(3)g'(3)}{(g(3))^2} = \frac{(4)(6) - (-6)(2)}{4^2}$$

$$= \frac{36}{16} = \boxed{\frac{9}{4}}$$

ii. If $h(x) = (f \circ g)(x)$ find $h'(3)$

$$h'(3) = f'(g(3)) \cdot g'(3)$$

$$= f'(4) \cdot g'(3) = 7 \cdot 2 = \boxed{14}$$

3. (20 points) Find $f'(x)$ for:

(a) $f(x) = \sqrt{1 - \cos^3 x}$

$$f'(x) = \frac{1}{2}\left(1 - \cos^3 x\right)^{-\frac{1}{2}} \left(-3\cos^2 x\right)(-\sin x)$$

(b) $f(x) = \sin(\tan x)$

$$f'(x) = \cos(\tan x) \cdot \sec^2 x$$

(c) $f(x) = (\arctan(2x))^3$

$$f'(x) = 3\left(\arctan(2x)\right)^2 \frac{1}{1+(2x)^2} \cdot 2$$

(d) If $f(x) = e^{3x}$, find $f^{2026}(x)$ the 2026th derivative.

$$f'(x) = 3e^{3x}$$
$$f''(x) = 9e^{3x}$$
$$f'''(x) = 27e^{3x}$$
$$\vdots$$

$$\vdots$$
$$f^{2026}(x) = 3^{2026} e^{3x}$$

4. (20 points) Find $\frac{dy}{dx}$ for:

(a) $y = e^{x^2} + 7^{e^x}$

$$\frac{dy}{dx} = 2x\, e^{x^2} + 7^{e^x} (\ln 7)\, e^x$$

(b) $y = \ln \sqrt[3]{(x^2 + e^x)(42x + \ln x)}$

$$= \frac{1}{3}\left[\ln(x^2 + e^x) + \ln(42x + \ln x)\right]$$

$$\frac{dy}{dx} = \frac{1}{3}\left[\frac{2x + e^x}{x^2 + e^x} + \frac{42 + \frac{1}{x}}{42x + \ln x}\right]$$

(c) $y = (x^2 + 9)^{\sin x}$

$$\ln y = \ln (x^2+9)^{\sin x}$$

$$= \sin x \ln(x^2 + 9)$$

$$\frac{1}{y}\frac{dy}{dx} = \sin x \frac{2x}{x^2+9} + \cos x \ln(x^2+9)$$

$$\frac{dy}{dx} = (x^2+9)^{\sin x}\left[\sin x \frac{2x}{x^2+9} + \cos x \ln(x^2+9)\right]$$

(d) $y = 7x^3 \cosh(6x)$

$$\frac{dy}{dx} = 7x^3 \sinh 6x \cdot 6 + 21x^2 \cosh(6x)$$

5. (20 points)

(a) For the curve

$$x^3 + y^2 = 5xy - 3$$

i. Find the slope of the tangent line at the point $(1, 1)$.

$3x^2 + 2y\frac{dy}{dx} = 5x\frac{dy}{dx} + 5y \Big|_{(1,1)}$

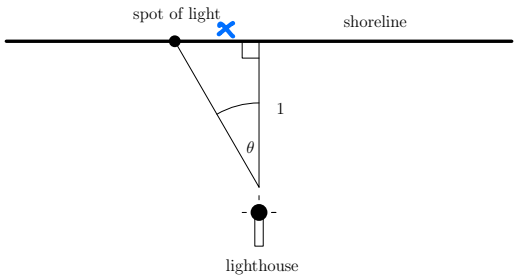
$3 + 2\frac{dy}{dx} = 5\frac{dy}{dx} + 5 \qquad -2 = 3\frac{dy}{dx}$

$\boxed{\frac{dy}{dx} = -\frac{2}{3}}$

ii. Find the equation of the tangent line at $(1, 1)$.

$\boxed{y - 1 = -\frac{2}{3}(x-1)}$

(b) The beacon of a lighthouse 1 km from a straight shore revolves 5 times per minute and shines a spot of light on the shore (see figure below).



How fast is the spot of light moving along the shore when $\theta = \frac{\pi}{6}$?

$\tan\theta = \frac{x}{1}$

$\sec^2\theta \frac{d\theta}{dt} = \frac{dx}{dx}$

$\frac{d\theta}{dt} = \frac{5 \text{ REV}}{\text{MIN}} \cdot \frac{2\pi \text{ RAD}}{\text{REV}} = 10\pi/\text{MIN}$

$\sec^2\frac{\pi}{6} = \frac{1}{\cos^2\frac{\pi}{6}} = \frac{1}{\left(\frac{\sqrt{3}}{2}\right)^2} = \frac{4}{3}$

$\frac{dx}{dt} = \sec^2\left(\frac{\pi}{6}\right)(10\pi)$

$= \frac{4}{3}(10\pi)$

$= \boxed{\frac{40\pi}{3} \text{ KM/MIN}}$