Math 121 Test 1

EF:

September 15, 2026

Name KEY

1	
2	
3	
4	
5	
Total	

Directions:

1. No books, notes or adding an extra second to the end of the game. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.

2. You should show your work and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.

3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.

4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

5. This test has 5 problems.

1. (20 Points)

(a) Write the inequality $|2x - 3| < 5$ in the form $a < x < b$.

$-5 < 2x - 3 < 5$

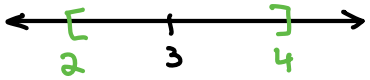
$-2 < 2x < 8$

$-1 < x < 4$

$a = -1$

$b = 4$

(b) Suppose $|x - 3| \leq 1$, what is the largest vale of $|x + 2|$?



$2 \leq x \leq 4$

$4 \leq x + 2 \leq 6$

$|x + 2| \leq 6$

6

(c) Find the equation of the line perpendicular to $2x + 3y = 5$ that goes through $(3, 2)$.

$3y = -2x + 5$

$y = -\frac{2}{3}x + \frac{5}{3}$

$m = -\frac{2}{3}$

$m_\perp = \frac{3}{2}$

$y - 2 = \frac{3}{2}(x - 3)$

(d) Find $(f \circ g)(x)$ for $f(x) = \sin x$ and $g(x) = x + \frac{1}{x}$

$(f \circ g)(x) = f(g(x)) = \sin(g(x))$

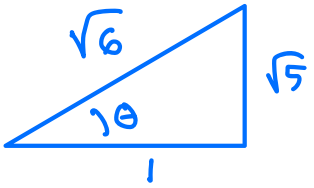
$= \sin\left(x + \frac{1}{x}\right)$

2. (20 points)

(a) If $\tan\theta = \sqrt{5}$ and $\sin\theta < 0$, find:

i. $\cot\theta$

$\cot(\theta) = \frac{1}{\sqrt{5}}$



ii. $\cos\theta$

$\cos\theta = -\frac{1}{\sqrt{6}}$

(b) Solve for x: $2^{x^2-2x} = 8$

$2^{x^2-2x} = 2^3$ $\qquad x^2 - 2x = 3$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x = 3, -1$

(c) Solve for x: $\log_2(x-2) + \log_2(x-3) = 1$

$\log_2(x-2)(x-3) = 1$

$x^2 - 5x + 6 = 2$

$x^2 - 5x + 4 = 0$

$(x-1)(x-4) = 0$

$x = 1, 4$ $\qquad x \neq 1$

$x = 4$ ONLY

3. (20 points)

(a) $\lim_{x\to 0} \dfrac{(x+a)^2 - a^2}{x}$

$= \lim_{x\to 0} \dfrac{x^2 + 2ax + a^2 - a^2}{x}$

$= 2a$

(b) $\lim_{x\to 5} \dfrac{x-5}{\sqrt{x+4}-3}$ $\cdot \left(\dfrac{\sqrt{x+4}+3}{\sqrt{x+4}+3}\right)$

$= \lim_{x\to 5} \dfrac{(x-5)(\sqrt{x+4}+3)}{(x+4-9)} = 3+3$

$= 6$

(c) $\lim_{x\to 3^-} \dfrac{x^2 - x - 6}{|x^2 - 9|}$

$= \lim_{x\to 3^-} -1\dfrac{(x-3)(x+2)}{|(x-3)(x+3)|} = -\dfrac{5}{6}$

(d) $\lim_{x\to 0} x^2 e^{\sin\left(\frac{1}{x}\right)}$

$-1 \le \sin\left(\frac{1}{x}\right) \le 1$

$\frac{1}{e} \le e^{\sin\left(\frac{1}{x}\right)} \le e$

$\frac{x^2}{e} \le x^2 e^{\sin\left(\frac{1}{x}\right)} \le x^2 e$

$\frac{x^2}{e} \to 0$ and $x^2 e \to 0$

$\lim_{x\to 0} x^2 e^{\sin\left(\frac{1}{x}\right)} = 0$

4. (20 points)

(a) If $f(x) = \dfrac{2x}{x-4}$, find $f^{-1}(x)$

$y = \dfrac{2x}{x-4}$

$x = \dfrac{2y}{y-4}$ $\quad x(y-4) = 2y$ $\quad y(x-2) = 4x$

$xy - 4x = 2y$

$xy - 2y = 4x$

$\boxed{f^{-1}(x) = \dfrac{4x}{x-2}}$

(b) $\lim\limits_{x\to 0} \dfrac{3x + 4\sin 3x}{\sin 5x - x\cos 2x}$

$= \lim\limits_{x\to 0} \dfrac{\frac{3x}{x} + \frac{4\sin 3x}{3x}\cdot 3}{\frac{5\sin 5x}{5x} - \frac{x\cos 2x}{x}} = \dfrac{3+12}{5-1} = \boxed{\dfrac{15}{4}}$

(c) $\lim\limits_{x\to -\infty} \dfrac{\sqrt{3x^2 + 2x + 6}}{5 - 3x}$

$= \boxed{\dfrac{\sqrt{3}}{3}}$

(d) Find the value of a so that $f(x)$ is continuous if

$$f(x) = \begin{cases} \dfrac{x^2-4}{x-2} & \text{for } x < 2 \\ 3ax - 8 & \text{for } x \geq 2 \end{cases}$$

$\lim\limits_{x\to 2^-} f(x) = \lim\limits_{x\to 2^-} \dfrac{(x-2)(x+2)}{(x-2)} = 4$

$\lim\limits_{x\to 2^+} f(x) = \lim\limits_{x\to 2^+} 3ax - 8 = 6a - 8$

$6a - 8 = 4 \quad 6a = 12 \quad \boxed{a = 2}$

5. (20 points) Indicate whether the following statements are true or false by circling the appropriate letter. A statement which is sometimes true and sometimes false should be marked false.

a) If $f(x)$ is continuous at $x = c$ then $\lim_{x \to c} f(x) = f(c)$. **T** **F**

b) If $f(x)$ is not defined at $x = c$ then $\lim_{x \to c} f(x)$ does not exist. **T** **F**

c) If $\lim_{x \to 1} f(x) = 5$, then $\lim_{x \to 1^+} f(x) = 5$. **T** **F**

d) If $f(1) = -2$ and $f(4) = 5$, then there is a c with $1 < c < 4$ where $f(c) = 0$. **T** **F**

e) If $f(x)$ is a polynomial and $f(1) = -2$ and $f(4) = 5$, then there is a c with $1 < c < 4$ where $f(c) = 0$. **T** **F**

f) If $f(x)$ is a polynomial and $f(1) = -2$ and $f(4) = 5$, then $f(x) < 6$ for all $1 \leq x \leq 4$. **T** **F**

g) If $\lim_{x \to 0} \dfrac{f(x)}{x} = 1$, then $\lim_{x \to 0} f(x) = 0$ **T** **F**

h) If $\lim_{x \to c} f(x) = L$, then there is an ϵ such that if $0 < |x - c| < 0.01$ then $|f(x) - L| < \epsilon$ **T** **F**

i) If $\lim_{x \to c} f(x) = L$, then there is a δ such that if $0 < |x - c| < \delta$ then $|f(x) - L| < 5$ **T** **F**

j) $1 + 1 = 3$ **T** **F**