

Math 121 Test 1

September 19, 2023

EF:

--

1	
2	
3	
4	
5	
Total	

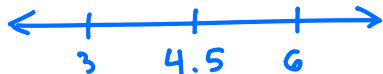
Name KEY

Directions:

1. No books, notes or dropping the rental car keys to the bottom of the lake. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.
2. You should show your work and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.
3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.
4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.
5. This test has 5 problems.

1. (20 Points)

- (a) If the inequality $3 \leq x \leq 6$ is written in the form $|x - a| \leq b$, what are a and b ?



$$|x - 4.5| \leq 1.5$$

$$a = 4.5$$

$$b = 1.5$$

- (b) What is the domain of $f(x) = x^{-4} + (x - 1)^{-3}$?

$$x \neq 0, 1$$

- (c) Find the slope, y -intercept, and x -intercept of the line with equation $y = 2x - 3$

$$\text{SLOPE} = 2, \quad x\text{-INT} = \frac{3}{2}$$

$$y\text{-INT} = -3$$

- (d) Find $(f \circ g \circ h)(1)$ for $f(x) = 2x - 1$, $g(x) = 3x$ and $h(x) = x^2 + 1$

$$h(1) = 2$$

$$g(2) = 6$$

$$f(6) = 11$$

$$(f \circ g \circ h)(1)$$

$$= f(g(h(1)))$$

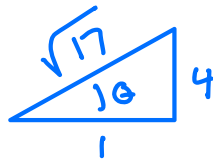
$$= f(g(2))$$

$$= f(6)$$

$$= 11$$

2. (20 points)

(a) Find $\cos \theta$ if $\cot \theta = \frac{1}{4}$ and $0 \leq \theta < \pi/2$.



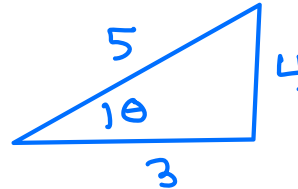
$$\cos \theta = \frac{1}{\sqrt{17}}$$

(b) Find the exact value of $\sin(\arctan(\frac{4}{3}))$

$$\theta = \arctan\left(\frac{4}{3}\right)$$

$$\tan(\theta) = \frac{4}{3}$$

$$\sin \theta = \frac{4}{5}$$



(c) Solve for x : $4^{-x} = 2^{x+1}$

$$2^{-2x} = 2^{x+1}$$

$$-2x = x+1$$

$$-3x = 1$$

$$x = -\frac{1}{3}$$

(d) Solve for x : $2 \ln x - \ln(x+1) = \ln 4 - \ln 3$

$$\ln \frac{x^2}{(x+1)} = \ln\left(\frac{4}{3}\right)$$

$$\frac{x^2}{x+1} = \frac{4}{3}$$

$$3x^2 = 4x + 4$$

$$x = 2, \quad \cancel{-\frac{2}{3}}$$

$$3x^2 - 4x - 4 = 0$$

$$(x-2)(3x+2) = 0$$

3. (20 points)

$$(a) \lim_{x \rightarrow 1} \frac{x^2 - 2x + 1}{x^3 + 1} = \frac{0}{2} = 0$$

$$(b) \lim_{a \rightarrow b} \frac{a^2 - 3ab + 2b^2}{a - b} = \lim_{a \rightarrow b} \frac{\cancel{(a-b)}(a-2b)}{\cancel{(a-b)}}$$

$$= b - 2b = -b$$

$$(c) \lim_{x \rightarrow 4^-} \frac{x - 4}{\sqrt{x^2 - 8x + 16}} = \lim_{x \rightarrow 4^-} \frac{x - 4}{\sqrt{(x-4)^2}}$$

$$= \lim_{x \rightarrow 4^-} \frac{x - 4}{|x - 4|} = -1$$

$$(d) \lim_{x \rightarrow 5} \frac{x - 5}{\sqrt{x + 4} - 3} \cdot \frac{\sqrt{x + 4} + 3}{\sqrt{x + 4} + 3}$$

$$= \lim_{x \rightarrow 5} \frac{\cancel{(x-5)}\sqrt{x+4} + 3}{\cancel{x+4} - 9} = 6$$

4. (20 points)

(a) If $f(x) = \frac{3x+2}{5x-1}$, find $f^{-1}(x)$

$$y = \frac{3x+2}{5x-1} \quad x = \frac{3y+2}{5y-1} \quad 5xy - x = 3y + 2$$

$$5xy - 3y = x + 2 \quad y = \frac{x+2}{5x-3} \quad f^{-1}(x) = \frac{x+2}{5x-3}$$

(b) $\lim_{x \rightarrow 0} \frac{\tan(4x)}{x \sec x}$

$$= \lim_{x \rightarrow 0} \frac{\sin 4x}{4x} \cdot \frac{1}{\cos(4x)} \cdot \frac{\cos x}{1} \cdot 4 = 4$$

$\swarrow \quad \cdot \quad \swarrow \quad \cdot \quad \swarrow \quad \cdot \quad \swarrow$

(c) $\lim_{x \rightarrow \infty} \frac{\sqrt{9x^4 + 3x + 2}}{2x^2 - 5x + 10}$

$$= \frac{3}{2}$$

(d) Find the value of a and b so that $f(x)$ is continuous.

$$f(x) = \begin{cases} x+1 & \text{if } x < 1 \\ ax+b & \text{if } 1 \leq x \leq 2 \\ 3x & \text{if } x > 2 \end{cases}$$

$$\lim_{x \rightarrow 1^-} x+1 = 2$$

$$\lim_{x \rightarrow 1^+} ax+b = a+b$$

$$\lim_{x \rightarrow 2^-} ax+b = 2a+b \quad \lim_{x \rightarrow 2^+} 3x = 6$$

$$\begin{aligned} a+b &= 2 \\ 2a+b &= 6 \end{aligned}$$

$$a = 4 \quad b = -2$$

5. Indicate whether the following statements are true or false by circling the appropriate letter. A statement which is sometimes true and sometimes false should be marked false.

- a) If $\lim_{x \rightarrow 3} f(x) = L$, then $L = f(3)$. T **F**
- b) If $\lim_{x \rightarrow c} f(x) = 2$, then $\lim_{x \rightarrow c} (f(x))^3 = 8$. **T** F
- c) If $f(x)$ is continuous on $[2, 3]$ and $f(2) = 4$ and $f(4) = 16$, then $f(3) = 9$ T **F**
- d) If $f(x)$ is continuous on $[0, 5]$ and $f(0) = -1$ and $f(2) = 5$, then there is $0 < c < 5$ such that $f(c) = 0$. **T** F
- e) If $f(x) = \frac{x^2 - 5x + 6}{x - 3}$ then $f(x)$ is not defined at $x = 3$ **T** F
- f) If $\lim_{x \rightarrow c} f(x) = 5$ and $\lim_{x \rightarrow c} g(x) = 1$ then $\lim_{x \rightarrow c} (2f(x) - 3g(x)) = 7$ **T** F
- g) If $2x - 1 \leq f(x) \leq x^2$ for $0 \leq x \leq 3$ then $\lim_{x \rightarrow 1} f(x) = 1$ **T** F
- h) If $\lim_{x \rightarrow 3} f(x) = 4$, there is a number $\delta > 0$ such that if $0 < |x - 3| < \delta$, then $|f(x) - 4| < 0.01$ **T** F
- i) If $\lim_{x \rightarrow 3} f(x) = 4$, there is a $\epsilon > 0$ such that if $|f(x) - 4| < \epsilon$ then $0 < |x - 3| < 0.01$ T **F**
- j) $1 + 1 = 2$ **T** F