Math 121 Test 1

September 17, 2019

EF:

1	
2	
3	
4	
5	
6	
Total	

Name KEY

Directions:

1. No books, notes or drawing comical pictures of your Chemistry Instructor. You may use a calculator to do routine arithmetic computations. You may *not* use your calculator to store notes or formulas. You may not share a calculator with anyone.

2. You should show your work and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.

3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.

4. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.

5. This test has 6 problems.

1. (20 Points)

(a) Which of the following intervals corresponds to the set of all x satisfying $|x-3|<5$?

(a) $(-8,-2)$ (b) $[-8,-2]$ (c) $(-8,2)$ (d) $[-8,2]$

(e) $(-2,8)$ (f) $[-2,8]$ (g) $(2,8)$ (h) $[2,8]$

$-5 < x - 3 < 5$

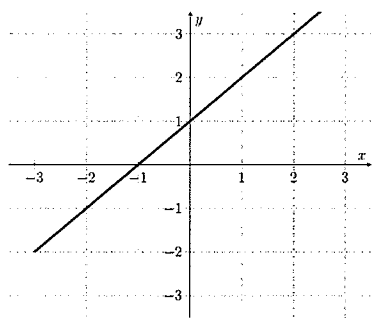
$-2 < x < 8$

(b) What is the domain of $f(x)=\ln\sqrt{-x}$?

(a) $(-\infty - 1)$ (b) $(-\infty,-1]$ (c) $(-\infty,0)$ (d) $(-\infty,0]$

(e) $(0,\infty)$ (f) $[0,\infty]$ (g) $(1,\infty)$ (h) $[1,\infty]$

(c) For the function $y=f(x)$ graphed below, what is $f^{-1}(1)$?



$f(0) = 1$

$f^{-1}(1) = 0$

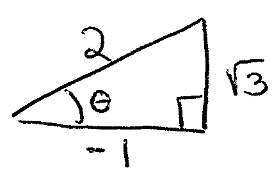
(a) -2 (b) -1 (c) $-1/2$ (d) 0 (e) $1/2$ (f) 1 (g) 2

(h) None of the above, $f(x)$ does not have an inverse.

(d) Suppose $\sin x = \frac{\sqrt{3}}{2}$ and $\cos x < 0$. What is $\tan x$?

(a) $-\sqrt{3}$ (b) -1 (c) $-\frac{\sqrt{3}}{4}$ (d) $-\frac{1}{2}$

(e) $-\frac{1}{2}$ (f) $\frac{\sqrt{3}}{4}$ (g) 1 (h) $\sqrt{3}$



2. (20 points)

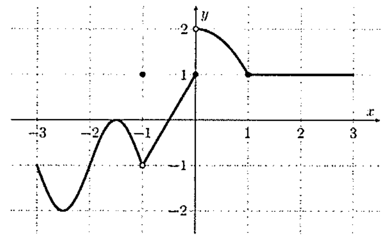
(a) If $y = \log_3 x$, which of the following is equal to $\ln x$?

(a) e^{3y} (b) $\log_3(\ln y)$ (c) $\ln(3^y)$ (d) $\frac{\ln 3}{\ln y}$

$x = 3^y$

$\ln x = \ln(3^y)$

(b) The function $y = f(x)$ is graphed below. For which values of c does $\lim_{x \to c} f(x)$ **not** exist?



(a) -1 only (b) 0 only (c) 1 only (d) -1 and 0 only

(e) 2 (f) 0 and 1 only (g)-1, 0, 1 only

(h) None of the above: the limits exists for all c

(c) If $f(x)$ satisfies the inequality $2x - 1 \leq f(x) \leq x^2$, then what is the $\lim_{x \to 1} f(x)$?

(a) -2 (b) -1 (c) 0 (d) 1 (e) 2

(f) The limit does not exist.

(g) The limit cannot be determined from the information given.

(d) What is $\lim_{x \to 2} \frac{x^2 - x - 2}{x - 2}$ $= \lim_{x \to 2} \frac{\cancel{(x-2)}(x+1)}{\cancel{(x-2)}}$

(a) -3 (b) -2 (c) -1 (d) 0 (e) 1 (f) 2 (g) 3

(h) None of the above, the limit does not exist.

3. (20 points)

(a) Find the inverse of the function $f(x) = \frac{4x-1}{2x+3}$

$x = \frac{4y-1}{2y+3}$ $\quad 2xy + 3x = 4y - 1$ $\quad 3x + 1 = 4y - 2xy$

$y = \frac{3x+1}{4-2x}$ $\quad f^{-1}(x) = \frac{3x+1}{4-2x}$

(b) $\lim_{x\to 4} \frac{\sqrt{x}-2}{3-\sqrt{5+x}} \cdot \frac{\sqrt{x}+2}{\sqrt{x}+2} \cdot \frac{3+\sqrt{5+x}}{3+\sqrt{5+x}}$

$= \lim_{x\to 4} \frac{\overset{-1}{\cancel{(x-4)}}(3+\sqrt{5+x})}{\cancel{9-(5+x)}(\sqrt{x}+2)} = \frac{-6}{4} = -\frac{3}{2}$

(c) $\lim_{x\to 1} \frac{3^{2x}+3^x-12}{3^x-3} = \lim_{x\to 1} \frac{\cancel{(3^x-3)}(3^x+4)}{\cancel{(3^x-3)}}$

$= 7$

(d) $\lim_{x\to\infty} \frac{8x^4-12x+1023}{4x^4-1228x^2+654x+15} = \frac{8}{4} = 2$

4. (20 points)

(a) $\lim_{x\to 0} \dfrac{x + x\cos x}{\sin x \cos x}$

$$= \lim_{x\to 0} \frac{x(1+\cos x)}{\sin x \cos x} = 1 \cdot 2 = 2$$

(b) $\lim_{x\to 3^-} \dfrac{x^2 - 2x - 3}{|x-3|}$ $= \lim_{x\to 3^-} \dfrac{\overset{-1}{\cancel{(x-3)}}(x+1)}{\cancel{|x-3|}}$

$$= -4$$

(c) Find the value of a and b so that $f(x)$ is continuous if

$$f(x) = \begin{cases} -x & \text{for } x < -1 \\ ax + b & \text{for } -1 \le x < 1 \\ x^2 + 2 & \text{for } x \ge 1 \end{cases}$$

$\lim_{x\to -1^+} f(x) = 1$ $\quad \lim_{x\to -1^-} f(x) = -a + b$ $\quad -a + b = 1$

$a + b = 3$

$\lim_{x\to 1^-} f(x) = a + b$ $\quad \lim_{x\to 1^+} f(x) = 3$ $\quad 2b = 4$

$\boxed{a = 1 \quad b = 2}$

(d) Find a value of $\delta > 0$, so that $|(2x-2) - 4| < 0.01$, if $|x-3| < \delta$.

$|2x - 6| < 0.01$

$|2(x-3)| < 0.01$

$|x - 3| < .005$ $\quad \delta = .005$ OR SMALLER

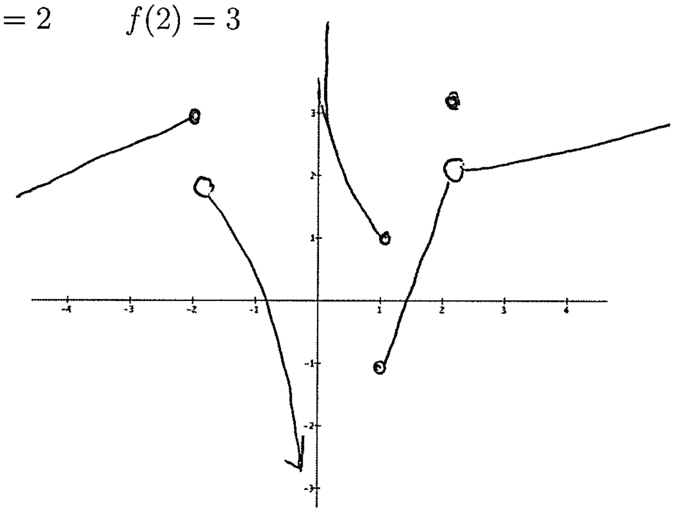
5. (10 points) On the axes below, sketch (if possible) the graph of a function that meets all of the following criteria:

$\lim_{x \to -2^-} f(x) = 3$ $\quad\lim_{x \to -2^+} f(x) = 2$

$\lim_{x \to 0^-} f(x) = -\infty$ $\quad\lim_{x \to 0^+} f(x) = +\infty$

$\lim_{x \to 1^-} f(x) = 1$ $\quad\lim_{x \to 1^+} f(x) = -1$

$\lim_{x \to 2} f(x) = 2$ $\quad f(2) = 3$



6. (10 points) Indicate whether the following statements are true or false by circling the appropriate letter. A statement which is sometimes true and sometimes false should be marked false.

a) The function $f(x) = \dfrac{x+1}{x^2 - x - 2}$ is continuous at $x = -1$. **T** **F**

b) If $f(x) = \sin x$ and $g(x) = x^2$ then $(f \circ g)(x) = \sin^2 x$ **T** **F**

c) If $\lim_{x \to 1} f(x) = 5$, then $\lim_{x \to 1^+} f(x) = 5$ **T** **F**

d) If $\lim_{x \to 5} f(x) = 2$, then $\lim_{x \to 5} (f(x))^3 = 8$ **T** **F**

e) If $\lim_{x \to 0} \dfrac{f(x)}{x} = 1$, then $\lim_{x \to 0} f(x) = 0$ **T** **F**