Directions:

1. No books, notes or stolen pizzas. You may use a calculator to do routine arithmetic computations. You may not use your calculator to store notes or formulas. You may not share a calculator with anyone.

2. You should show your work, and explain how you arrived at your answers. A correct answer with no work shown (except on problems which are completely trivial) will receive no credit. If you are not sure whether you have written enough, please ask.

3. You may not make more than one attempt at a problem. If you make several attempts, you must indicate which one you want counted, or you will be penalized.

4. Numerical experiments do not count as justification. For example, computing the first few terms of a series is not enough to show the terms decrease. Computing the first few partial sums of a series is not enough to show the series converges or diverges. Plugging in numbers is not enough to justify the computation of a limit.

5. On this test, explanations count. If I can’t follow what you are doing, you will not get much credit.

6. You may leave as soon as you are finished, but once you leave the exam, you may not make any changes to your exam.
1. (10 points) Find the limit of the following sequences:

(a) \( \left\{ \frac{2n^2 - 3n + 1}{5 - 7n^2} \right\}_{n=1}^{\infty} \)

(b) \( \left\{ n^2 - \sqrt{n^4 + 3n} \right\}_{n=1}^{\infty} \)

2. (10 points) Determine whether the following series converge or diverge. If they converge find the sum.

(a) \( \sum_{n=0}^{\infty} \left( \frac{1}{4^n} + \left( \frac{2}{3} \right)^n \right) \)

(b) \( \sum_{n=0}^{\infty} \left( \frac{1}{1 + e^{-n}} \right) \)
3. (10 points) For each of the following series, determine if it converges or diverges. For each test you use, you must name the test, perform the test, and state the conclusion you reached from that test.

(a) \[ \sum_{n=1}^{\infty} \frac{\arctan n}{n^2 + 1} \]

(b) \[ \sum_{n=1}^{\infty} n^{-2} \sin \left( \frac{1}{n} \right) \]

4. (10 points) For each of the following series, determine if it converges or diverges. For each test you use, you must name the test, perform the test, and state the conclusion you reached from that test.

(a) \[ \sum_{n=1}^{\infty} \frac{\sqrt{n + 3}}{n \sqrt{n + 2}} \]

(b) \[ \sum_{n=1}^{\infty} \frac{(2n)!}{4!(n!)^2(2n + 1)} \]
5. (10 points) For each of the following series, determine if the following series converge absolutely, converge conditionally, or diverge. For each test you use, you must name the test, perform the test, and state the conclusion you reached from that test.

(a) \[ \sum_{n=2}^{\infty} (-1)^n \frac{1}{\sqrt[n]{n}(\sqrt[n]{n} - 1)} \]

(b) \[ \sum_{n=1}^{\infty} (-1)^n \left( \frac{1}{1 + e^{-n}} \right) \]

6. (10 points) Consider the power series:

\[ \sum_{n=1}^{\infty} \frac{(2x - 4)^n}{4^n \sqrt[n]{n}} \]

(a) Where is the power series centered?

(b) Find the radius of convergence.

(c) Find the interval of convergence. [Hint: Check the endpoints.]
7. (10 points) For \( f(x) = \sin(x^3) \)

(a) Write out the first four nonzero terms for the Maclaurin series for \( f(x) \).

(b) Find \( f^{(600)}(0) \) (the 600th derivative of \( f(x) \) at \( x = 0 \)).

8. (10 points)
For the area of region inside \( r = 8 \) and outside \( r = 8 \cos 5\theta \).
Fill in the boxes.

\[
A = \int \left(\frac{2^2}{2} \right) d\theta - \int \left(\frac{2^2}{2} \right) d\theta
\]
9. (10 points) For the curve

\[ c(t) = \left( t - \frac{t^3}{3}, t^2 \right) \]

(a) Find the speed at time \( t = 1 \).

(b) Find the length of the curve for \( 0 \leq t \leq 2 \).

10. (10 points) True or False, indicate whether the following statements are true or false by circling the appropriate letter. A statement which is sometimes true and sometimes false should be marked false.

   a) A convergent series with some positive terms is absolutely convergent.  
      [T F]

   b) There are two ways to represent a rectangular point in polar coordinates.  
      [T F]

   c) If \( \sum_{n=1}^{\infty} a_n^2 = 4 \) then \( \sum_{n=1}^{\infty} a_n = 2 \)  
      [T F]

   d) The vertex for the parabola \( y = \frac{x^2}{4} - x + 9 \) is at \( (2, 8) \)  
      [T F]

   e) The foci for the ellipse \( \left( \frac{x - 3}{\sqrt{2}} \right)^2 + \left( \frac{y + 2}{\sqrt{7}} \right)^2 = 1 \) are \( (2, 0) \) and \( (2, -6) \)  
      [T F]
FORMULA PAGE

$$\sin^2 \theta + \cos^2 \theta = 1$$
$$\tan^2 \theta + 1 = \sec^2 \theta$$
$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$
$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$
$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$
$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\sin^2 x = \frac{1 - \cos 2x}{2}$$
$$\cos^2 x = \frac{1 + \cos 2x}{2}$$
$$\sin 2x = 2 \sin x \cos x$$
$$\cos 2x = 2 \cos^2 x - 1$$
$$(\sin x)' = \cos x$$
$$(\cos x)' = -\sin x$$
$$(\tan x)' = \sec^2 x$$

$$(\sec x)' = \sec x \tan x$$
$$(\csc x)' = -\csc x \cot x$$
$$(\cot x)' = -\csc^2 x$$

$$(e^x)' = e^x$$
$$(\ln x)' = \frac{1}{x}$$

$$(\arcsin x)' = \frac{1}{\sqrt{1 - x^2}}$$
$$(\arctan x)' = \frac{1}{1 + x^2}$$
$$(\text{arcsec } x)' = \frac{1}{|x| \sqrt{x^2 - 1}}$$

$$(\sinh x)' = \cosh x$$
$$(\cosh x)' = \sinh x$$

$$\sinh x = \frac{e^x - e^{-x}}{2}$$
$$\cosh x = \frac{e^x + e^{-x}}{2}$$

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

$$f(c) = \frac{1}{b - a} \int_a^b f(x) \, dx$$

$$\int \sec x \, dx = \ln |\sec x + \tan x| + C$$
$$\int \sec^3 x \, dx = \frac{1}{2} [\sec x \tan x + \ln |\sec x + \tan x|] + C$$
$$\int \csc x \, dx = \ln |\csc x - \cot x| + C$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \ldots = \sum_{k=0}^{\infty} \frac{x^k}{k!}, \text{ for all } x$$
$$\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} + \ldots = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k+1}}{(2k+1)!}, \text{ for all } x$$
$$\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} + \ldots = \sum_{k=0}^{\infty} (-1)^k \frac{x^{2k}}{(2k)!}, \text{ for all } x$$

$$\frac{1}{1 - x} = 1 + x + x^2 + x^3 + x^4 + \ldots = \sum_{k=0}^{\infty} x^k, \text{ for } -1 < x < 1$$
$$1 + 1 = 2$$
<table>
<thead>
<tr>
<th>NAME</th>
<th>STATEMENT</th>
<th>COMMENTS</th>
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<tbody>
<tr>
<td>Geometric Series</td>
<td>$\sum_{k=0}^{\infty} ar^k$</td>
<td>Converges if $</td>
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<td></td>
<td>Diverges if $</td>
<td>r</td>
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<tr>
<td>$p$-series</td>
<td>$\sum_{k=1}^{\infty} \frac{1}{k^p}$</td>
<td>Converges if $p &gt; 1$, diverges if $p \leq 1$</td>
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<tr>
<td>Divergence Test</td>
<td>If $\lim_{k \to \infty} a_k \neq 0$, the $\sum_{k=1}^{\infty} a_k$ diverges.</td>
<td>If $\lim_{n \to \infty} a_k = 0$, $\sum_{k=1}^{\infty} a_k$ may or may not converge.</td>
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<td>Integral Test</td>
<td>Let $\sum_{k=1}^{\infty} a_k$ be a series with positive terms, and</td>
<td>Uses this test when $f(x)$ is easy to integrate.</td>
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<td>let $f(x)$ be the function that results when $k$ is replaced by $x$ in the formula for $a_k$. If $f$ is decreasing and continuous for $x \geq 1$, then $\sum_{k=1}^{\infty} a_k$ and $\int_{1}^{\infty} f(x) , dx$ both converge or both diverge.</td>
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<td>Comparison Test</td>
<td>Let $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ be a series with positive terms such that if $a_k &lt; b_k$ and $\sum_{k=1}^{\infty} b_k$ converges then $\sum_{k=1}^{\infty} a_k$ converges or if $b_k &lt; a_k$ and $\sum_{k=1}^{\infty} b_k$ diverges then $\sum_{k=1}^{\infty} a_k$ diverges</td>
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<td>Limit Comparison Test</td>
<td>Let $\sum_{k=1}^{\infty} a_k$ and $\sum_{k=1}^{\infty} b_k$ be a series with positive terms such that $\lim_{k \to \infty} \frac{a_k}{b_k} = \rho$ If $0 &lt; \rho &lt; \infty$, then both series converge or both diverge.</td>
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| Ratio Test                  | Let $\sum_{k=1}^{\infty} a_k$ be a series with positive terms and suppose $\lim_{k \to \infty} \frac{a_{k+1}}{a_k} = \rho$  
  a) Series converges if $\rho < 1$.  
  b) Series diverges if $\rho > 1$.  
  c) No conclusion if $\rho = 1$.  
  Try this test when $a_k$ involves factorials or $k$-th powers. |                                               |
| Root Test                   | Let $\sum_{k=1}^{\infty} a_k$ be a series with positive terms and suppose $\lim_{k \to \infty} \sqrt[k]{a_k} = \rho$  
  a) Series converges if $\rho < 1$.  
  b) Series diverges if $\rho > 1$.  
  c) No conclusion if $\rho = 1$.  
  Try this test when $a_k$ involves $k$th powers |                                               |
| Alternating Series Test     | Let $\sum_{k=1}^{\infty} a_k$ be a series with alternating terms if $\lim_{k \to \infty} a_k = 0$ and $|a_k| > |a_{k+1}|$ the series converges conditionally  
  This test applies only to alternating series. |                                               |