The Statistical Properties of Competitive Districts: What the Central Limit Theorem Can Teach Us about Election Reform

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Any examination of newspaper editorials and reform group positions will indicate that the most commonly perceived problem with U.S. elections at the moment is that they are rarely competitive. This absence of competition has prompted recent proposals in several states, most prominently in California and Ohio, to reform the redistricting process to increase the frequency of competitive elections. These propositions failed, but that is unlikely the end of such attempts. The reason for these proposals is obvious. Political education in the U.S. indoctrinates us at a young age to believe that competition is good. In economics, market competition provides social benefits, and, by analogy, political competition must provide similar benefits. This argument is frequently made explicitly, such as by Schumpeter (1942), and it has its roots as far back as the often-assigned *Federalist Papers* #10 and #51. Of course, the economic definition of competition has little to do with the political definition of a competitive election. Broadly speaking, there are two economic definitions of competition: (1) competitive behavior, which generally means innovation, and (2) competitive market structures, which means many buyers, many sellers, and perfect information. In contrast, a competitive election generally means one in which the candidates' vote shares are roughly equal, or similarly, one in which each candidate has a roughly 50% chance of winning. The only obvious relationship between these definitions is that we might equate vote shares with market shares, but the benefits of a competitive market accrue regardless of whether sellers have equal market shares—they simply must charge the same price for the same goods, as determined by the intersection of the supply and demand curves. In fact, in a competitive market, each firm should have such a small market share that no firm is a price-giver, and the idea of an election for a single-member district in which there are so many candidates that no candidate has a significant vote share seems untenable. Alternatively, we might draw an analogy between pressure on sellers to charge the same price for the same good and pressure on candidates toward policy convergence, but the idea of all candidates offering the same bundle of policies makes many people uncomfortable as well. So, the analogy between the efficiency of a competitive market and the benefits of a competitive election falls apart. Of course, Schumpeter's (1942) argument was that political leaders should compete for votes by innovating with respect to policy based on the first definition of competition, but again, that means that the economic analogy has little to say about whether elections should be uncertain and decided by narrow margins. The fact that we happen to use the same word for different circumstances does not logically allow us to say the following: because competition is good in the marketplace, close elections are good. Free market economics cannot be used to defend the drawing of competitive districts without straining the analogy beyond its limits, regardless of how intuitively appealing the analogy is.

However intuitive it may seem that we should pursue redistricting plans with more competitive districts, research on the process has shown that there are several negative consequences of doing so. There are a variety of arguably worthy goals to pursue when drawing district lines, and fulfilling one goal generally means sacrificing another. For example, one might argue that it is important for a redistricting plan to produce partisan representation proportionate to the popular vote, but proportionality must be sacrificed in order to draw competitive districts. This relationship can be seen empirically since we see disproportionality associated with the frequency of competitive elections (Butler and Cain 1992), and there are a variety of theoretical explanations for this pattern (see, for example, Niemi and Deegan 1978; Niemi 1982). Persily (2002) even argues that non-competitive elections have positive consequences for representation. Brunell (2006) argues that minimizing the number of competitive districts maximizes voter satisfaction, and Buchler (2005) argues that an unbiased but non-competitive redistricting plan maximizes ideological representation. Given these and other tradeoffs, it is not particularly clear why it is necessary for districts to be competitive. However, these arguments suffer largely from their counterintuitive nature, and are rejected by supporters of redistricting reform not simply on the merits, but also on the basis of that intuition. Competition just seems intuitively appealing, largely because of the positive connotations of the word itself in a free market system. However, intuition is subjective, and "counterintuitive" does not mean "wrong."

This paper has two related purposes: (1) to make these counterintuitive arguments about the negative consequences of competitive elections more intuitive by applying basic statistical reasoning, and (2) to demonstrate the wide applicability of basic statistical concepts that are frequently considered and taught at too abstract a level. In particular, this paper will rely on applications of the Central Limit Theorem.[1] There are three obvious applications of the Central Limit Theorem to the concept of competitive elections and the question of their associated benefits. First, having more competitive elections increases the likelihood of declaring the wrong candidate the winner. Second, having more competitive elections necessarily promotes disproportionality. Finally, and most paradoxically, having more competitive districts, which is not the same as having competitive elections, actually runs the risk of promoting entrenchment and reducing the

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likelihood of a change in control of Congress.

Confidence in Election Results

What went wrong in Florida in the 2000 presidential election to lead to protracted court battles and disputed election results? Votomatic-style punch card systems and the butterfly ballot design are the obvious culprits, but they would have been irrelevant if the election had not been so close. From a statistical perspective, what went wrong in Florida was that the election was too competitive. The closer an election, the less confident we can be with the declaration of the winner. Regardless of the turnout rate, the voting population is still a sample, and thus there is some probability that the winner in our sample is not actually the candidate preferred by a majority in the population. However, for the sake of argument, let us focus simply on measuring the preferences of voters regardless of whether they are a biased sample of the population.

Consider a hypothetical House election with 99,999 voters. Suppose 50,000 individuals vote for Candidate A, and 49,999 vote for Candidate B. For the sake of convenience, suppose that whenever a vote is counted, it is counted correctly, but that there is a 1% vote disqualification rate. For example, when voters use a generally reliable system such as an optical scan voting system, perhaps 1% of votes are not counted for various reasons (see, for example, CalTech/MIT 2001; and Brady et al. 2001). If that is the case, then any given count of the votes is a 99% sample (hopefully) without replacement. It may be easier to think of this problem as follows: suppose we randomly select 1,000 votes (1%) for disqualification to avoid sampling a large proportion of the population without replacement. If we select at least 500 votes for Candidate B to disqualify, Candidate A will win the count, and if we select at least 501 votes for Candidate A to disqualify, Candidate B will win. What is the sampling distribution of the number of disqualified votes that would be votes for Candidate B? The sampling distribution of the proportion of disqualified votes that would be votes for Candidate B would be a normal distribution with mean of $\mu = .5$, and variance of $\sigma^2 = .25/1000$: $N(\mu = .5, \sigma^2 = 2.5 * 10^{-4})$. Since this is a normal distribution which is symmetric around .5 and the variance is extremely small, the probability that any given count will require an immediate recount is 1, and the probability that either candidate will win any given count is essentially .5. Thus, the probability that the same candidate wins the original count and the initial recount is only .5, meaning that there is a 50% chance that the first count and the recount produce different winners. Furthermore, there is only a 25% chance that Candidate A will win both the initial count and the recount. Thus, there is a 75% chance that at least one of the counts will be a mistake. The inevitable court battles that would occur would make those of Florida in 2000 seem calm and friendly by comparison. Given the results of any set of counts, nobody could be more than 50% confident that the declared winner is the "true" winner. Despite the fact that Candidate A is the "true" winner, each candidate has a .5 probability of being declared the winner. The closer an election is, the higher the likelihood that the result will be "wrong." In the extreme, little could be more harmful to public confidence in election results than close elections. How close do we really want our elections to be? Do we want to argue that elections should be decided by a margin of victory between 1% and 5%, no more and no less? Such claims would lead us down a slippery slope in which we argue about an arbitrary upper *and lower* bound for acceptable margins of victory. Such debates would be a poor guide for policy-making.

Proportionality of Election Results

As the mid-term elections of 2006 approached, predictions about the results came in two varieties. First, those who examined nationwide polls found that a strong majority of respondents claimed to prefer Democratic control of Congress in every major survey. However, given that there were very few House races expected to be competitive, at the level of individual House elections, Republicans seemed to be in a stronger position than the national numbers suggested because in order to gain control of either the House or the Senate Democrats had to sweep so many of the seats that were in play. Until just before the election, it was not at all clear that the Democrats would win control despite their strong advantage in nationwide surveys. The primary culprit for this disparity was the redistricting process, and criticism of redistricting focused on the scarcity of competitive districts. The common argument was that there were too few competitive districts, which insulated the Republican majority from popular trends. While the insulation was not sufficient to protect their majority in 2006, advocates of redistricting reform were bothered by the perceived obstacle the lack of competitive districts presented. There are two types of concerns here: proportionality and entrenchment. This section will focus on proportionality, and the following section on entrenchment. With respect to proportionality, the supposed problem was that Republicans might have held more seats than voters nationwide would prefer, and that this disproportionality results from lack of competition in congressional elections. As discussed earlier, though, the more competitive seats there are, the less proportionality there is between the partisanship of representatives and the partisanship of the voters, and there is a very simple explanation based on the statistical properties of competitive elections.

Suppose there are N legislative seats, and the population is evenly divided. If we draw N evenly divided districts, then each district will be potentially competitive. Suppose each party has a .5 probability of winning a potentially competitive seat. What is the sampling distribution of the proportion of seats that Party A will win? Again, we turn to the Central Limit Theorem. The distribution will be a normal distribution with a mean of .5 and a variance of .25/N: $N(\mu = .5, \sigma^2 = .25/N)$. The probability that a draw from this distribution will be exactly .5 ironically becomes very small as the number of districts increases because the distribution becomes more smooth. On average, though, we expect either party to win 50% of the seats, and as the number of districts increases, the expected deviation from 50% decreases. Thus, the plan is unbiased and increasingly efficient as the number of districts increases, but there is still inefficiency. It is highly likely that there will be at least some small disproportionality, although the direction of that disproportionality will be determined by random chance.

However, suppose we draw homogeneous districts such that half of the districts consist entirely of voters from Party A, and half of the districts consist entirely of voters from Party B. Since each district is non-competitive, the proportion of seats that Party A wins will be .5 by necessity. If districts are homogeneous, the overall results will be proportional to the voters' preferences deterministically. If districts are competitive, the *expectation* of the aggregate election results will be unbiased, but that is only because errors in either direction are equally likely. If districts are homogeneous, the results are just as unbiased, but perfectly efficient, so the expected disproportionality is actually greater with more competitive districts (related effects are discussed in more detail in Buchler Forthcoming). Competitive districts produce disproportionate election results based simply on the statistical properties of competitive elections. Allowing uncertainty in the election process permits error to creep into election results, and we must be willing to accept those errors if we want uncertainty. If we want competitive elections, we must sacrifice proportionality. We cannot count on competitive elections to promote proportionality.

Competition and Entrenchment

Non-competitive districts may promote proportionality as long as the redistricting plan is unbiased, and this result is well-established in the redistricting literature. However, the idea makes many uncomfortable because it suggests the possibility of entrenchment. If districts are non-competitive, one party may hold a majority indefinitely, even if the preferences of voters shift. A swing ratio of 10 may be bad, but isn't a swing ratio of 0 bad too? Don't we need competitive districts in order to make sure neither party becomes entrenched in power? Again, basic statistical principles show the flaw in this reasoning. Suppose 217 House seats are safe Democratic Seats, 217 seats are safe Republican seats, and one seat is marginal. If safe seats are won deterministically by the party controlling the seat and marginal seats are coin tosses, then control of the chamber will depend entirely on the single marginal seat, and each party will have a 50% chance of winning control of the chamber even though there is only a single competitive seat. Similarly, suppose there are n marginal seats where n is an odd number, $(435 - n)/2$ Republican seats, and $(435 - n)/2$ Democratic seats. If each party has a .5 probability of winning any given marginal seat, then the sampling distribution of the proportion of marginal seats that either party will win is $N(\mu = .5, \sigma^2 = .25/n)$. Thus, the probability that either party will control the chamber is still .5. Control of the chamber can be contested with equal probabilities regardless of the number of marginal districts. What matters is not how many competitive seats there are, but how many safe seats there are for each party relative to each other. A lower probability of a shift in control of the chamber comes from one party having more safe seats than the other, not from the absence of competitive districts.

Of course, if the number of safe seats on each side is unequal, then the probability that either party controls the chamber will depend partially on the number of competitive districts. Suppose the majority party holds M safe seats, the minority party holds m safe seats, and there are n contested seats where $218 > M > m > n$ (that way, it is at least mathematically possible for either party to win control of the chamber). The number of seats the majority party wins will be a draw from a $N(\mu = M + .5n, \sigma^2 = .25n)$ distribution. What would happen if we increase n? Would that increase the likelihood of the minority party taking control of the chamber? That depends on how we increase n. Suppose we create two more competitive seats by taking away one safe seat from each party and balancing them out (which is simple enough if the partisan advantage in any safe seat is the same margin for each party). That would leave the expected seat share of each party unchanged, but increase the variance of the sampling distribution of the *number* of seats each party wins, which increases the probability that the minority party will win enough seats to take control of the chamber. Now, we must answer two questions. First, what are the side effects of doing so, and second, is there a method of increasing the probability of a takeover without such side effects?

With respect to the first question, there is an obvious side effect. The normal distribution is symmetric, and just as we increase the likelihood that the disadvantaged party will take control of the chamber, we also increase the likelihood that the advantaged party will become even more entrenched, which may have both policy consequences and consequences for future elections. The policy consequences come from the possibility that a more decisive majority within the chamber might allow the majority party to pursue more extreme policies without having to make concessions either to the minority party or the moderate elements of the majority party. The consequences for future elections are a bit less obvious, and again require us to return to the basic statistical properties of competitive elections.

A decisive majority itself may lead to entrenchment. Since incumbents have advantages beyond the drawing of district lines (financial advantages, name recognition advantages, the ability to provide constituent services and particularized benefits, etc.), once a competitive seat is won by one party, the probability that the other party will take over that district might be less than .5 even if the district has an even partisan balance. Now, let us consider another hypothetical example. Suppose the probability that an incumbent loses in a balanced district is .25. If each party has 217 safe seats, then there is one competitive seat. Control of the chamber is decided by that single seat, and each party has a .5 probability of winning control when the competitive seat is open. Once that party has control, the minority party has a 25% chance of taking over in the next election because they have a 25% chance of beating the incumbent in the pivotal district. However, what if each party had 216 safe seats, and there were three competitive seats? Each party has a .5 probability of winning control of the chamber when these three seats are open, but there is also a 25% chance that one of the two parties will win all three competitive seats (each party has a $.5^3 = .125$ probability of winning all three seats, so there is a .25 probability than one of the two parties will win all three competitive seats). If that happens, in order for control of the chamber to change again in the next election, the minority party must win two out of three competitive seats, but they only have a 25% chance at any one seat. There are three ways they can win precisely two seats (1 and 2, 1 and 3, 2 and 3). The probability of each of those outcomes individually is .046875 ($.25 * .25 * .75 = .046875$), so the probability of one of those outcomes occurring is .140625 ($3 * .046875 = .140625$). There is also a .015625 probability that the minority party wins all three competitive seats ($.25^3 = .015625$). Thus, if one party wins all three competitive seats, there is a $.140625 + .015625 = .15625$ probability that the minority party will win control in the subsequent election. However, if there had been only one competitive seat, that probability would have been fixed at .25. Increasing the number of competitive seats also creates the risk of greater entrenchment, and if that happens, the probability that control of the chamber will change again will be lower than it would have been with fewer competitive seats since having fewer competitive seats eliminates the possibility of such entrenchment.

To be fair to the competitive plan, though, there is a 75% chance that neither party will win all three open seats, and if the majority party holds only two of the competitive seats, the probability that the minority party will gain control again is .34375, which is slightly more tedious to demonstrate.[2] Thus, in the competitive plan, there is a 75% chance that the minority party will have a 34% chance of gaining control in the second election, and a 25% chance that the minority party will have a 16% chance of gaining control. Thus, we can calculate the probability that the minority party will win control in the second election as follows: (.75 * .34375) + (.25 * .15625) = .296875. So, *prior to the first election* with three competitive districts, there is a 30% chance that the minority party will gain control in the second election, which is higher than the 25% chance with only one competitive district. However, in order to get that higher *initial* probability of a change in control in the second round, we must accept the risk of the updated probability declining dramatically, which may have policy consequences in addition to electoral consequences. So, we might argue that minimizing the number of competitive districts is the risk-averse approach to preventing entrenchment.

Paradoxically, increasing the number of competitive seats carries with it a risk of greater entrenchment, which can reduce the probability that control of the chamber will change again in updated probabilities. While counterintuitive, the principle is quite simple if we think about it in terms of basic statistics. Having more competitive districts increases the number of seats that are within reach of either party, which creates the possibility of a more decisive majority. If we draw a large number of competitive districts, random chance may allow one party to win a larger majority than they should win, and in fact, a larger majority than they could win in an unbiased but non-competitive plan. Furthermore, a competitive district does not guarantee a competitive election, and once a party gains such a decisive majority through random chance, it becomes that much harder for the minority party to win control of the chamber in the future. Thus, there is a danger to having a large number of competitive districts—it actually risks entrenchment.

Now, given that potentially dangerous side effect of drawing more competitive districts, we turn to the second question. Is there a less dangerous way to increase the likelihood of a change in chamber control? The obvious answer now is yes—equalize the number of safe seats for each party and minimize the number of competitive seats. In the short run, balancing the number of safe seats is crucial because if there is an imbalance, increasing the number of competitive races without changing the balance of safe seats will cause the probability of a change in control of the chamber to asymptotically approach .5, but never reach .5. If there is a balance in the number of safe races, the probability of either party controlling the chamber is precisely .5 regardless of the number of competitive seats. Thus, in the short run, balancing the number of safe seats is a better way to reduce entrenchment than increasing the number of competitive seats. Furthermore, if we are going to balance the number of safe seats, we might as well minimize the possibility of either party becoming *deeply* entrenched by gaining a decisive majority, and we minimize that possibility by reducing the number of competitive seats rather than increasing it. The best way to allow for a change in party control without risking further entrenchment is to balance the number of safe seats and minimize the number of competitive seats. These consequences of redistricting plans seem much more intuitive in the context of the basic statistical properties of competitive elections.

Concluding Remarks

The fact that a competitive election is more likely to produce an error in the declared winner should be obvious, but advocates of increased competition in congressional elections do not address the point. One might reasonably think that a 40-point margin of victory is too high. However, if we take that position, then what is an acceptable margin of victory? A .001-percentage-point margin of victory may be too close, but is that better or worse than a 40-point margin? The consequence of taking that line of reasoning is that we wind up with some arbitrarily defined range of acceptable levels of competition such that more or less competition is unacceptable. Such an arbitrarily defined range of acceptable election results would be a poor guide for policy-making.

The issue of confidence in election results should not be controversial, although one must wonder how important it is relative to other concerns, particularly since an election result has to be very close before it becomes an issue, and even competitive congressional districts rarely have election results that close. However, competitive elections have other negative consequences based on the statistical properties of uncertain elections. It has already been established that competitive congressional districts produce partisan representation that is disproportionate to the preferences of voters, but homogenous congressional districts produce proportionate results. The arguments presented here merely simplify the argument by presenting it in the context of the Central Limit Theorem.

The most controversial element here is likely to be the argument that increasing the number of competitive congressional districts might paradoxically increase the risk of majority party entrenchment. However, this makes intuitive sense if we think about elections based on statistical theory. If each side has an equal number of safe seats, then control of the chamber is a coin toss regardless of the number of competitive seats. Furthermore, a large number of competitive seats runs the risk of one side gaining a decisive majority. If a competitive district does not guarantee a competitive election because of incumbency advantages, such a decisive majority would simply make it even more difficult for the minority to win control later. In the long run, drawing more competitive districts might reduce the likelihood of a change in control of the chamber. At the very least, the best way to ensure that control of the House is uncertain is to equalize the number of safe seats on each side. Furthermore, if we do that, we might as well reduce the number of competitive seats in order to minimize the risk of deep entrenchment.

These arguments are far from an exhaustive examination of the meaning of competition and the statistical properties of competitive districts. However, they demonstrate that basic statistical principles have a great deal to say about election reform, and they demonstrate that by relying on such principles as the Central Limit Theorem, we can make a set of previously counterintuitive claims seem much more intuitive. Competitive districts may seem intuitively appealing, but their mathematical properties are frequently not.

Notes

1. For reference purposes, the Central Limit Theorem is briefly restated here. If we draw a random sample of n observations from a population with an expectation of μ and a variance of σ^2, and we compute the average of our sample of n observations, the Central Limit Theorem tells us that the sampling distribution of that average is $N(\mu, \sigma^2/n)$ distribution.

2. The probability that the minority party will retain its own seat and win precisely one of the majority party's seats is .75 * .75 * .25 = .140625, and there are two ways this can

happen, so the probability that the minority party retains its own seat and wins one of the majority party's seats is .28125. The minority party could also win control if all competitive seats change hands, and the probability of that occurring is $.25 * .25 * .25 = .015625$. Finally, the minority party can win control by winning all three competitive seats, and the probability of that is $.75 * .25 * .25 = .046875$. Thus, the probability that the minority party will win control of the chamber if the majority holds two out of three competitive seats is $.28125 + .015625 + .046875 = .34375$.

References

Brady, Henry E., Justin Buchler, Matt Jarvis, and John McNulty. 2001. *Counting All The Votes*. Berkeley: Survey Research Center/Institute of Governmental Studies, UC Berkeley.

Brunell, Thomas. 2006. "Rethinking Redistricting: How Drawing Uncompetitive Districts Eliminates Gerrymanders, Enhances Representation, and Improves Attitudes toward Congress." *PS: Political Science and Politics* 39 (January): 77–85.

Buchler, Justin. 2005. "Competition, Representation and Redistricting: The Case against Competitive Congressional Districts." *Journal of Theoretical Politics* 17 (4): 431–63.

———. Forthcoming. "The Social Suboptimality of Competitive Elections." *Public Choice*.

Butler, David, and Bruce E. Cain. 1992. *Congressional Redistricting: Comparative and Theoretical Perspectives*. New York: McMillan.

CalTech/MIT Voting Technology Project. 2001. "A Preliminary Assessment of the Reliability of Existing Voting Equipment." Version 1: February 1.

Niemi, Richard G., and John Deegan, Jr. 1978. "A Theory of Political Districting." *American Political Science Review* 72 (4): 1304–23.

Niemi, Richard G. 1982. "The Effects of Districting on Tradeoffs among Party Competition, Electoral Responsiveness, and Seats-Votes Relationships." In *Representation and Redistricting Issues*, eds. Bernard Grofman, Arend Lijphart, Robert B. McKay, and Howard A. Scarrow. Lexington, KY: Lexington Books.

Persily, Nathaniel. 2002. "In Defense of Foxes Guarding Henhouses: The Case for Judicial Acquiescence to Incumbent-Protecting Gerrymanders." *Harvard Law Review* 116 (2): 649–83.

Schumpeter, Joseph. 1942. *Capitalism, Socialism and Democracy*. New York: Harper and Row.