Public Choice (2011) 148: 1–19
DOI 10.1007/s11127-010-9643-4

The proximity paradox: the legislative agenda and the electoral success of ideological extremists

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Received: 1 May 2009 / Accepted: 24 March 2010 / Published online: 9 April 2010
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Abstract This paper presents a new approach to spatial models of legislative elections in which voters have preferences over the bundles of roll call votes implied by candidate locations rather than over the locations themselves. With such preferences, voters with single-peaked, symmetric preferences and perfect information can sincerely prefer a distant candidate to a more proximate candidate. Moreover, negative agenda control in Congress makes such preference orderings inevitable, so party agenda control can allow majority party extremists to defeat more centrist minority party candidates. The model has implications for theories of parties in Congress, and spatial modeling more broadly.

Keywords Elections · Congress · Agenda · Voting · Proximity

Conventional spatial models assume that voters derive utility directly from the location of the winning candidate. That allows us to treat an election as a choice between candidate locations as though those locations were specific policies. However, a choice between two legislative candidates is not a choice between their locations in the policy space. Instead, it is a choice between the bundles of roll call votes implied by their locations in the policy space. So, this paper proposes a model in which voters have preferences, not over the location of the winning candidate, but over the bundles of roll call votes implied by the location of the winning candidate. With such preferences, even voters with single-peaked, symmetric preferences and perfect information can sincerely prefer the more distant candidate to the more proximate candidate—a phenomenon this paper calls a "proximity paradox." This paper then presents an application of the "proximity paradox" by examining how party control over the legislative agenda can influence legislative election results. Cox and McCubbins (1993, 2005) demonstrate that the majority party controls the legislative agenda, in part by prohibiting floor votes when the majority party leadership would lose (negative agenda control). This paper demonstrates that by doing so, the majority party will create a "proximity

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paradox" that will allow majority party candidates to defeat minority party candidates, even when the minority party candidates are closer to their district median voters, and even when voters behave sincerely based on single-peaked, symmetric preferences and perfect information.

1 Spatial models with violations of proximity voting

The objective of this paper is to demonstrate that the structure of the legislative agenda can allow ideological extremists to defeat more centrist candidates. Such election results can only occur if voters do not always vote for the most proximate candidate. There have been a wide variety of explanations for violations of proximity voting, including the directional model (Rabinowitz and MacDonald 1989), the inclusion of a valence dimension (Enelow and Hinich 1982, or more recently, Zakharov 2009), and the possibility of asymmetric utility functions (Persson and Tabellini 2000).[1] However, the model in this paper is most closely related to policy output models. Austen-Smith (1981, 1984, 1986) and Persson and Tabellini (2000), for example, argue that voters might consider the process of legislative bargaining that occurs after an election to generate policy outputs, and if they do so, they may have strategic incentives to vote for a more distant candidate over a more proximate candidate. Similarly, Lacy and Paolino (1998) argue that voters must discount candidates' campaign promises because the tension between the legislative and executive branches will prevent winning candidates from actually implementing their locations in the policy space. The critical insight in these models is that we cannot simply assume that the winner of an election unilaterally provides constituents with policies equivalent to her location in the policy space. This paper advances that line of reasoning in three critical ways. First, this paper will connect the candidate location process directly to the process of casting roll call votes. While the roll call voting process is arguably the most critical means by which a winning candidate implements her location in the policy space, that process has received precious little attention from spatial theories of *elections*.[2] Second, this paper will connect spatial models of elections directly to one of the primary models of party influence in Congress—the cartel model (Cox and McCubbins 1993, 2005). Finally, this paper will generate violations of proximity voting when voters are sincere. It is trivially easy to generate violations of proximity voting when voters behave insincerely, as they do in Austen-Smith (1981, 1984, 1986), Persson and Tabellini (2000) and Lacy and Paolino (1998). However, this paper will present a model in which voters sincerely prefer distant candidates to more proximate candidates.

The argument will draw primarily on Schattschneider's (1960) conception of political conflict in which the struggle is to determine where lines of cleavage are placed, which is conceptually and mathematically identical to the process of structuring roll call votes. Suppose voters have preferences over the votes that legislators cast rather than over their locations in the policy space. If so, then even if voters are single-minded policy-seekers with single-peaked, symmetric preferences, they may *sincerely* prefer the bundle of votes offered by the more distant candidate to the bundle offered by the most proximate candidate.

[1] It should be noted, though, that even if voters have asymmetric utility functions, Persson and Tabellini (2000) show that some median voter results remain.

[2] Fowler and Frederking (2000) acknowledge the point, but pursue it in a different manner.

2 A citizen-candidate model with violations of proximity voting

This paper uses a citizen-candidate model (Osborne and Slivinski 1996; Besley and Coate 1997; Cadigan and Janeba 2002; Klumpp 2007) under a plurality election rule. All citizens are potential candidates who simultaneously decide whether or not to become candidates. Then, an election is held in which voters choose between all announced candidates under a plurality rule. Citizen-candidate models assume that candidates cannot credibly commit to insincere locations, so if they choose to run, they run on their ideal points, and implement their ideal points if elected. Where this model breaks from previous citizen-candidate models is that the winner of the election will not simply provide all constituents with policies equivalent to her ideal point. Instead, she will implement her ideal point by casting roll call votes on the basis of her ideal point given an agenda imposed by the majority party leadership. This paper will use a citizen-candidate model because the substantive result depends on elections in which both candidates diverge from the median voter's location, as they do in reality (Ansolabehere et al. 2001). Of course, there are a variety of other approaches that can generate divergent general elections, such as allowing party activists to push party platforms away from the center (as in Aldrich 1983a, 1983b; Poutvaara 2003). However, this paper does not address why divergence occurs, but instead how voters respond to it. Thus, the substantive results would be identical regardless of why divergence occurs.

The structure will be as follows. First, the paper will demonstrate that an individual voter with single-peaked, symmetric preferences can prefer a bundle of roll call votes offered by a distant candidate to a bundle associated with a more proximate candidate. Then, it will demonstrate that negative agenda control makes such preference orderings inevitable. It will then demonstrate how these results aggregate to permit a candidate further from the median voter to defeat a candidate closer to the median voter. Then, it will describe the two-candidate Nash equilibria of the model, and show that such paradoxical election results occur in equilibrium. Thus, majority party agenda control creates non-Downsian election results. Finally, the paper will discuss empirical predictions, and broad implications for spatial theory.

2.1 Assumptions

(I) Quadratic loss utility functions for policy. Each citizen has a quadratic loss utility function for policy $U_j(p) = -(p - i_j)^2$ where i_j is citizen j's ideal point. Thus, each actor has a single-peaked, symmetric utility function. By retaining the symmetry assumption, the model is biased in favor of proximity voting, but despite that, violations of proximity voting will follow.

(II) Perfect information. Citizens know the distribution of ideal points within the constituency. When the election occurs, each citizen knows the precise location of each candidate's ideal point. Furthermore, citizens know the locations of the *status quo* and *alternative* points for the roll call votes called in the previous session of Congress. The model will work equally well whether the voters know every roll call vote, or simply the subset of votes on high profile issues that receive media attention. The latter is obviously more realistic, but it is important to note that the model will work equally well, even if we make the highly unrealistic assumption that voters know the *status quo* and *alternative* points for *all* roll call votes. This is important because previous models have explained non-Downsian election results with imperfect information. Thus, by assuming that voters are *hyper*-informed, we bias the model in favor of proximity voting.

(III) Costs and benefits of candidate entry. If a citizen chooses to become a candidate and enter the election, she pays a cost $C > 0$. If she then wins, she receives a reward $R > 0$.

(IV) Retrospective evaluation of roll call votes. For incumbents, citizens compute the utility they would receive from the incumbent's reelection by calculating the total utility they received from the roll call votes the incumbent cast in the previous session. For challengers, citizens perform the same calculation using the roll call votes that the challenger would have cast if she had been in office. Since we have assumed that voters know the challenger's location and the *status quo* and *alternative* points for the votes in the previous session, voters can perform this calculation for incumbents and non-incumbents alike. Again, it is unimportant for the model whether voters know every roll call vote, or simply the subset of high profile votes. The math will be identical either way.

The viability of retrospective evaluation depends on how consistent the agenda is from one session of Congress to the next. If the agenda varies dramatically, this method will yield poor estimates of the utility of electing any given candidate. So, Appendix C discusses prospective evaluation of roll call votes. The same mathematical principles that generate the proximity paradox apply to prospective analysis, although majority party control of the legislative agenda would cease to be the mechanism producing it– campaign dynamics would produce the paradox.

(V) Sincere voting. Each citizen will vote for the candidate with a bundle of roll call votes providing her with the highest utility. Assumptions IV and V demonstrate where this model breaks conceptually from policy output models such as Austen-Smith (1981, 1984, 1986), Lacy and Paolino (1998), and Persson and Tabellini (2000). Voters calculate utility from roll call votes regardless of whether or not those votes affect actual policy output, and vote accordingly. There are two compelling reasons for this approach. The first is methodological. It is trivially easy to generate violations of proximity voting if voters care only about their representatives' votes when those votes are pivotal, such as in a move-the-median game or a balancing model. By assuming that voters calculate utility based on each roll call vote cast, the model is biased as strongly as possible in favor of proximity voting.

The second reason is more substantive. How should we expect voters to respond to past shirking?[3] Should we expect voters to forgive a shirked vote as long as their representative's vote was not pivotal, or should we expect voters to punish shirking even when their representative's vote was not pivotal? Empirical models of electoral punishment (e.g., Canes-Wrone et al. 2002; Brady et al. 2007) show that incumbents are punished for shirking generally, which makes substantive sense. If an incumbent shirks on a significant vote, her opponent will almost certainly call attention to it, and the incumbent is unlikely to offer the "I wasn't pivotal" defense because she knows that to the degree that voters punish shirking, they are unlikely to consider that to be a legitimate excuse.[4] A policy output model would require voters to permit shirking as long as a representative's vote is not pivotal. Since that seems unrealistic, this model requires voters to consider pivotal and non-pivotal votes equally.

2.2 Sequence

Each citizen simultaneously decides whether or not to enter the election. Citizens who become candidates run on their own ideal points because they cannot credibly commit to in-

[3]The traditional definition of shirking in this context is as follows: a legislator shirks if she votes for her preferred policy option when her preferences conflict with her constituents' preferences.

[4]There are examples of candidates explaining *missed* votes with that defense. Most recently, John McCain's spokesperson explained the number of votes he missed while campaigning for President by saying, "He has not missed a single vote where his vote would have changed the outcome," (Fischer, *The Arizona Daily Star*, 5/23/07). However, I am unaware of any examples of candidates explaining shirked votes with the "I wasn't pivotal" defense. Jacobs and Shapiro (2000) suggest that there are many other viable strategies for handling unpopular votes.

sincere locations, as is the convention in citizen-candidate models. Each candidate pays the cost C. Each citizen evaluates the utility of each candidate's victory by retrospectively assessing how each candidate would have voted on past roll call votes, and then votes for the candidate offering the bundle of roll call votes that maximizes her utility. The winner is chosen by a simple plurality rule. Since voters are sincere, the model will rely on a simple pure strategy Nash equilibrium.

3 Violations of proximity voting for an arbitrary voter

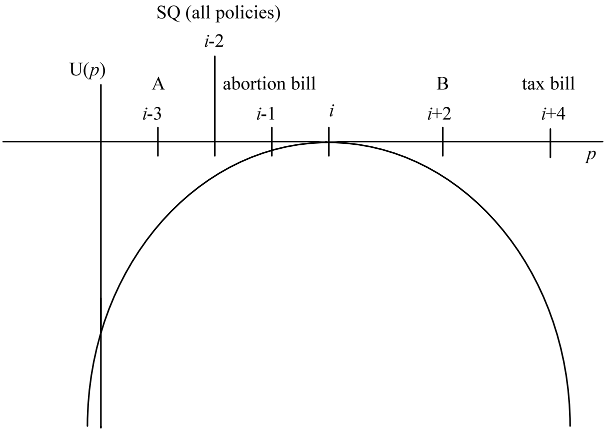
The first step is to demonstrate how an individual voter with single-peaked, symmetric preferences can prefer a candidate further from her ideal point to a candidate closer to her ideal point. Consider an arbitrary voter and two candidates, A and B, where $i_A < i_V < i_B$ and $i_B - i_V < i_V - i_A$. Thus, B is closer to the voter's ideal point than A. Regardless of that, we can easily construct a set of roll call votes such that the voter prefers A to B. Each roll call vote is a choice between a liberal alternative, l, and a conservative alternative, c where $l < c$. For the purposes of this model, it does not matter which is the status quo. Since each actor has a quadratic loss utility function, there is a cutpoint at $q = (l + c)/2$ such that if $q < i$, the actor prefers the conservative option, and if $q > i$, the actor prefers the liberal option. Suppose, first, that there is precisely one roll call vote with a cutpoint in the range (i_V, i_B), and that there are no roll call votes with cutpoints in the range (i_A, i_V). Thus, despite the fact that Candidate B is closer to the voter than Candidate A, the voter prefers the votes that Candidate A will cast because B will cast precisely one roll call vote with which the voter disagrees, and A will cast all votes consistently with the voter's preferences. Thus, we have our first demonstration of a *Proximity Paradox*—a preference ordering in which the voter *sincerely* prefers the more distant candidate to the more proximate candidate, even though the voter has single-peaked, symmetric preferences, perfect information, and has no non-policy considerations. Notice something important about this initial demonstration—it occurs based on the lack of roll call votes with cutpoints in a specific range. This will become critical because that is exactly what negative agenda control does.

However, it should be noted that the *Proximity Paradox* does not *require* roll call votes in a certain range to be prohibited. Consider another example. Suppose that there is precisely one vote, m, with a cutpoint q_m in the range (i_A, i_V), and that there is precisely one vote, n, such that q_n is in the range (i_V, i_B). Thus, A would cast all roll call votes consistently with the voter's preferences except vote m. B would cast all roll call votes except vote n in accordance with the voter's preferences. It does not follow from the candidates' proximity to the voter's ideal point that the voter prefers B to A. That, of course, depends on which roll call the voter would prefer to give up, m or n.

As a demonstration, suppose the voter's ideal point is i, while A and B are located at $i-3$ and $i+2$ respectively. Suppose, also, that all status quo points are at the same location: $i-2$. This need not be the case, but if we assume that the status quo point for all policies begins at the same location, the results will be generalizable to contexts in which the status quo points are *not* all located at the same point. Suppose Bill M would move abortion policy to a location of $i - 1$, and Bill N would move tax policy to a location of $i + 4$.

Since $q_m = i - 1.5$, A would vote against Bill M (and hence, for the abortion status quo), and B would vote for Bill M. Hence, B would vote consistently with the voter's preferences on the abortion bill, and A would vote inconsistently with the voter's preferences. On Bill N (the tax bill), the opposite would be true since $q_n = i + 1$. So, in order to determine which candidate the voter would sincerely prefer, we must calculate the voter's utility for

Fig. 1 A two-vote proximity paradox



the bundle of roll call votes that each candidate offers. The voter's total utility for A's votes is as follows:

$$U_v(i-2-i)+U_v(i-2-i)=-(-2)^2-(-2)^2=-4-4=-8 \tag{1}$$

The voter's total utility for B's votes is as follows:

$$U_v(i-1-i)+U_v(i+4-i)=-(-1)^2-(4)^2=-1-16=-17 \tag{2}$$

Clearly, the voter would prefer the utility she receives from A's votes to the utility she receives from B's votes, even though B's location is closer to her ideal point. Both A and B would cast a vote the voter does not like, so the voter must determine which lost vote would be more costly. Just because B's location is closer to her ideal point than A does not mean that the cost of her disagreement with B will be less than the cost of her disagreement with A. Thus, we have another *Proximity Paradox*, this time without any regions in which roll call votes are prohibited. However, this paper will focus on the effects of prohibited roll call votes because the purpose of the paper is to connect violations of proximity voting to party agenda control in the cartel model. So, this paper defines the *Proximity Paradox* as follows.

Definition A *Proximity Paradox* occurs in a spatial model when a policy-interested actor prefers a bundle of roll call votes associated with a candidate location further from her ideal point to a bundle of votes associated with a more proximate location.

Proposition 1 *For any voter with a single-peaked, symmetric and continuous utility function, if there are two candidates, A and B such that $i_A < i_V < i_B$ and $i_B - i_V < i_V - i_A$, then there exists some agenda of roll call votes such that the voter prefers the bundle of votes A would cast to the bundle of votes B would cast, even though B's location in the policy space is closer to the voter's ideal point than A's location. Thus, the assumption that a voter has a utility function that is both single-peaked and symmetric does not preclude a* Proximity Paradox.

Proof Suppose $i_A < i_V < i_B$ and $i_B - i_V < i_V - i_A$. If there are no roll call votes with cutpoints in the range (i_A, i_V), and there is at least one roll call vote with a cutpoint in the range (i_A, i_V), then the voter must prefer A's bundle of roll call votes to B's bundle of roll call votes because the voter will agree with every vote A would cast, and disagree with at least one that B would cast. However, as long as the voter's utility function is continuous, the *Paradox* does not require prohibited votes. Consider an agenda with one vote, m, with a cutpoint q_m in the range (i_A, i_V), and precisely one vote, n, such that q_n is in the range (i_V, i_B). Since the voter's utility function is continuous, as c_m converges to l_m, $U(c_m) - U(l_m)$ converges to 0. Thus, regardless of the utility differential between c_n and l_n, there exists some pair of alternatives, c_m and l_m, such that $U(c_m) - U(l_m) < U(l_n) - U(c_n)$. By simple algebra, it follows that $U(c_m) + U(c_n) < U(l_m) + U(l_n)$, meaning that the voter prefers A's bundle of votes to B's bundle of votes. Thus, whenever two candidates are located on opposite sides of a voter's ideal point, there will exist some array of roll call votes given which the voter prefers the bundle of votes offered by the more distant candidate to the bundle of votes offered by the most proximate candidate. □

We can generate violations of proximity voting, even with sincere voters who are single-minded policy seekers with single-peaked, symmetric preferences and perfect information. The key to the paradox is the distinction between preferences over candidate locations and preferences over the bundles of roll call votes which are implied by those locations. The paradox cannot occur if both candidates are on the same side of a voter's ideal point, or if one candidate is located at the voter's ideal point, as demonstrated in Appendix A. However, when candidates adopt polarized locations, as they do in modern elections (Ansolabehere et al. 2001), the critical swing voters are located between the two candidates, in which case the paradox can apply to critical voters.

4 Conditions of the proximity paradox

We must now describe the general circumstances under which the paradox occurs. Suppose $i_A < i_V < i_B$ and $i_B - i_V < i_V - i_A$. Suppose that there are M roll call votes with cutpoints in the range (i_A, i_V), and N roll call votes with cutpoints in the range (i_V, i_B). Roll call votes with cutpoints outside either range are irrelevant since neither candidate would vote inconsistently with the voter's preferences on them. A would provide the voter with the utility of the liberal option on each vote, and B would provide the voter with the utility of the conservative option on each vote. We can calculate the total utility of each set of roll call votes.

$$U(A) = \sum_{j=1}^{M} -(i - l_j)^2 + \sum_{k=1}^{N} -(i - l_k)^2 \tag{3}$$

$$U(B) = \sum_{j=1}^{M} -(i - c_j)^2 + \sum_{k=1}^{N} -(i - c_k)^2 \tag{4}$$

Then, it follows that the voter prefers the bundle of votes cast by A if the following condition is satisfied:

$$\sum_{j=1}^{M} -(i - l_j)^2 + \sum_{k=1}^{N} -(i - l_k)^2 > \sum_{j=1}^{M} -(i - c_j)^2 + \sum_{k=1}^{N} -(i - c_k)^2 \tag{5}$$

We cannot disprove this inequality because $-(i-l_k)^2 > -(i-c_k)^2\ \forall k$ even though $-(i-l_j)^2 < -(i-c_j)^2\ \forall j$, which is the generalized demonstration of the Paradox. Inequality (5) can be satisfied in one of two ways. First, if $M < N$, then there were more votes on which the voter disagrees with candidate B despite the fact that B's location is closer to the voter's ideal point. This is where prohibited votes in an agenda control model will become important. Alternatively, the utility differences in the second series on each side of the inequality may simply greater than the utility differences in the first series, in which case the cost of each disagreement with B is greater than the cost of each disagreement with A, as in the two-vote example presented earlier. In either case, the utility of A's roll call votes might be greater than the utility of B's votes.

This demonstration allows us to distinguish between arrays of roll call votes that permit the paradox, and arrays that prohibit the paradox. Suppose there are V voters with quadratic loss utility functions, where each voter has an ideal point, i_V. Suppose voters consider R roll call votes characterized by a conservative option, c_r, and a liberal option, l_r.[5]

Definition A *bounded subset of roll call votes* is a subset defined by some lower bound, L, and some upper bound, C, consisting of all votes for which $(l+c)/2 \in [L, C]$. Thus, if a candidate is located at L, and another candidate is located at C, the *bounded subset of roll call votes* defined by the range $[L, C]$ consists of all votes for which a voter located between the two candidates disagrees with precisely one candidate.

Definition For any given voter with ideal point, i, and *bounded subset of roll call votes* defined by $[L, C]$, suppose there are M_i roll call votes such that $(l_m + c_m)/2 \in [L, i]$, and N_i roll call votes such that $(l_n + c_n)/2 \in [i, C]$. R is a *symmetric set of roll call votes* if the following two conditions are met:

$$\sum_{j=1}^{M} -(i-l_j)^2 + \sum_{k=1}^{N} -(i-l_k)^2 < \sum_{j=1}^{M} -(i-c_j)^2 + \sum_{k=1}^{N} -(i-c_k)^2 \quad \forall i, L, C \text{ if } C-i < i-L \tag{6}$$

$$\sum_{j=1}^{M} -(i-l_j)^2 + \sum_{k=1}^{N} -(i-l_k)^2 > \sum_{j=1}^{M} -(i-c_j)^2 + \sum_{k=1}^{N} -(i-c_k)^2 \quad \forall i, L, C \text{ if } C-i > i-L \tag{7}$$

Definition R is an *asymmetric set of roll call votes* if there exists some value of i and some *bounded subset of roll call votes* defined by the range, $[L, C]$ where $C-i < i-L$ and:

$$\sum_{j=1}^{M} -(i-l_j)^2 + \sum_{k=1}^{N} -(i-l_k)^2 > \sum_{j=1}^{M} -(i-c_j)^2 + \sum_{k=1}^{N} -(i-c_k)^2 \tag{8}$$

or, some *bounded subset* $[L, C]$ where $C-i > i-L$ and:

$$\sum_{j=1}^{M} -(i-l_j)^2 + \sum_{k=1}^{N} -(i-l_k)^2 < \sum_{j=1}^{M} -(i-c_j)^2 + \sum_{k=1}^{N} -(i-c_k)^2 \tag{9}$$

[5] Again, this set may either be a complete enumeration of roll call votes from the previous session, or simply the subset of substantively important and high profile roll call votes. The math is identical either way.

Proposition 2 *If the set of roll call votes is symmetric, a Proximity Paradox cannot exist. If the set of roll call votes is asymmetric, then there exists some set consisting of a voter with ideal point i, and candidates A and B, such that the voter prefers the candidate located furthest from her ideal point, given the agenda of roll call votes.*

Proof Proposition 2 is true by construction since symmetric and asymmetric sets are defined specifically by whether or not they permit a Proximity Paradox. □

Whether or not voters will practice proximity voting depends on the structure of the legislative agenda. If the legislative agenda consisted of a symmetric set of roll call votes, then the candidate closest to the median voter would win deterministically. However, when faced with asymmetric sets of roll call votes, we will observe violations of proximity voting. At this point, though, the *Proximity Paradox* is merely a possibility. We must now demonstrate that it is more than just a possibility, and that majority party agenda control necessarily creates the paradox.

Proposition 3 *Suppose that the leader of the majority party is located at i_{PL}, and that the floor median in Congress is located at i_{FM} where $i_{PL} < i_{FM}$. If the majority party leader has the power to block floor votes, then the agenda will consist of an* asymmetric set of roll call votes, *producing a* Proximity Paradox *for some set of voters.*

Proof If the majority party leader can block a floor vote whenever she wants, then she will block any roll call vote that she would lose,[6] so there will be no roll call votes with cutpoints in the range (i_{PL}, i_{FM}). Thus, consider the *bounded subset of roll call votes* defined by a lower bound, L, where $L = i_{PL}$, and an upper bound of, C, where $C > i_{FM}$ but $C - i_{FM} < i_{FM} - L$. To continue with earlier notation, let M represent the subset of all roll call votes with cutpoints in the range (i_{PL}, i_{FM}), and let N represent the subset of all roll call votes with cutpoints in the range (i_{FM}, i_C). Since M is an empty set, it follows that:

$$\sum_{j=1}^{M} -(i-l_j)^2 + \sum_{k=1}^{N} -(i-l_k)^2 > \sum_{j=1}^{M} -(i-c_j)^2 + \sum_{k=1}^{N} -(i-c_k)^2 \quad \text{if } i_{PL} \le i \le i_{FM} \tag{10}$$

Thus, the legislative agenda is an *asymmetric set of roll call votes*. Next, suppose that there is a Democratic candidate located at L, and a Republican candidate located at C. Since Inequality 10 is true for any voter with an ideal point such that $0.5(L + C) < i_V \le i_{FM}$, and any such voter is closer to C than to L, it follows that negative agenda control creates a *Proximity Paradox* for any voter with an ideal point such that $0.5(L + C) < i_V \le i_{FM}$. □

The proof above demonstrates that negative agenda control necessarily creates a *Proximity Paradox* for voters with ideal points such that $0.5(L + C) < i_V \le i_{FM}$. This occurs because there are no roll call votes with cutpoints in the range, $[i_L, i_{FM}]$, so any candidate with a location in that range will cast all votes consistently with the preferences of every voter in that range, regardless of their proximity to any voter in that range. However,

[6]One might argue that the leader only has incentives to block such votes when she prefers the status quo because there is nothing lost by permitting a vote on an alternative that will lose anyway. However, if the majority party leader wants to prevent embarrassment, she will block all votes that she would lose, thus creating a "roll rate" of 0, in the terminology of cartel theory.

negative agenda control can create a *Proximity Paradox* even for voters outside that range. Suppose $i_A < i_{FM} < i_V < i_B$, and that $i_V - i_A > i_B - i_V$. Thus, the voter is still closer to B than to A. However, negative agenda control protects A from having to cast any vote with a cutpoint in the range (i_A, i_{FM}), which are votes with which the voter would disagree. That does not *necessarily* mean that the voter will prefer A to B, but it increases the likelihood that the voter will prefer A to B. So, negative agenda control necessarily creates a *Proximity Paradox* for some voters, and potentially creates such a paradox for many more.

5 The Proximity Paradox and aggregate election results

So far, we have seen that party agenda control will cause some individual voters to prefer distant candidates to proximate candidates, even when they have single-peaked, symmetric preferences. The next step is to show the effect of this paradox on aggregate election results. First, there is a caveat to the paradox.

Proposition 4 *If voters have single-peaked, symmetric preferences and receive utility from roll call votes rather than candidate locations, the location of the median voter's ideal point is a Condorcet winner in the election.*

Proof Suppose A is located at the median voter's ideal point, and B is located to the right of the median. By Lemma 1 (see Appendix A), the 50% of the voters who are to the left of A will either be indifferent between A and B, or prefer A, and by Lemma 2 (see Appendix A), the median must either be indifferent between A and B or prefer A. Thus, the electorate cannot prefer B to A. □

Proposition 4 is essentially a weak form of the Median Voter Theorem. However, as we shall see later, there are no two-candidate equilibria in which either candidate enters at the location of the median voter, so Proposition 4 has no relevance in equilibrium. Moreover, Proposition 4 is a razor's edge argument. Suppose $i_A < i_V < i_B$. Even if $(i_V - i_A)$ and $(i_B - i_A)$ are arbitrarily small, the existence of a single roll call vote with a cutpoint in the range (i_V, i_B) can cause the voter to prefer A to B, even if the voter is closer to B. Hence, Proposition 4 is little more than a phantom that only operates in an extremely restrictive context that cannot occur in equilibrium anyway. A more important derivation is Proposition 5:

Proposition 5 *Let* i_{MV} *represent the median voter's ideal point. Suppose four conditions are met*: (1) $i_{PL} < i_{MV} \leq i_{FM}$, (2) $i_{PL} < i_A < i_{FM}$, (3) $i_B > i_{FM}$, *and* (4) *there is at least one roll call vote with a cutpoint in the range* (i_{FM}, i_B). *If the majority party leader exercises negative agenda control, then A defeats B, even if B is closer to the median voter than A.*

Proof Since the majority leader will prevent any roll call vote with a cutpoint in the range (i_{PL}, i_{FM}), A will cast all votes consistently with the median voter's preferences. B will not. Thus, regardless of either candidate's proximity to the median voter, the median voter prefers A to B, as does every voter with an ideal point in the range (i_{PL}, i_{FM}). Furthermore, by Lemma 1, all voters to the left of i_{PL} must prefer A to B. Thus, A defeats B, regardless of which candidate is closest to the median voter's location. □

Proposition 5 demonstrates that negative agenda control permits non-centrist candidates of the majority party to defeat more centrist candidates of the minority party by protecting majority party members from having to cast electorally damaging votes. If the district median voter is located between the floor median and the majority party leader, as in Proposition 5, negative agenda control virtually guarantees victory to any majority party candidate over any minority party candidate. However, the *Proximity Paradox* can also protect non-centrist majority party members even when the district median voter is *not* located between the majority party leader and the floor median. Suppose that $i_{PL} < i_A < i_{FM} < i_{MV}$. Any vote with a cutpoint in the range (i_A, i_{MV}) would be electorally damaging to A, but negative agenda control limits the number of such votes that A would have to cast by blocking all roll call votes in the range (i_A, i_{FM}), and only forcing A to cast votes with cutpoints in the range (i_{FM}, i_{MV}). Negative agenda control provides no such protection for B and in fact, if the majority party leader is truly devious, she will *increase* the number of roll call votes with cutpoints in the range (i_{MV}, i_B), thus *maximizing* the effect of the median voter's disagreements with B. Hence, even if negative agenda control does not provide complete protection for majority party candidates, it provides at least partial protection, permitting them to defeat more centrist candidates of the minority party on the basis of sincere voting.

While Proposition 5 focuses on the relationship between the location of the floor median and the district median voter, it should be noted that when voters have preferences over bundles of roll call votes, the preference ordering for a group of voters is not always identical to the preference ordering of the median voter, as Appendix B demonstrates.

Hacker and Pierson (2005) suggest that the Republican Party managed to win elections prior to 2006 partially by setting the agenda in such a way as to hide the fact that they have moved away from the center. Similarly, Van Houweling (2003) argues that party control of legislative procedure permits Members to pursue their extreme policy preferences while hiding that extremism from their constituents. However, the model presented here shows that extremists can defeat centrists without any obfuscation or dishonesty. In this model, perfectly informed voters sincerely prefer the bundles of votes cast by extremists to the bundles of votes cast by moderates given the structure of the legislative agenda. The critical point is that preferences over candidate locations and preferences over the bundles of roll call votes implied by those locations are very different. Furthermore, there is no reason for policy-motivated voters to care about locations independently of the votes they imply. Hence, majority party agenda control can make it rational for perfectly informed voters with single-peaked, symmetric preferences to sincerely prefer their own extreme candidates to moderate candidates of the opposing party. Majority party agenda control creates non-Downsian election results.

This observation provides important insight into cartel theory. According to cartel theory (Cox and McCubbins 1993, 2005), parties strive to create electoral advantages for themselves by creating a brand name that will appeal to voters. The analysis presented so far has demonstrated that the agenda control techniques that follow from cartel theory create inherent electoral advantages on their own simply by protecting majority party incumbents from having to cast votes that would risk alienating potential swing voters. There need not be a cohesive brand name for the majority party to retain its status through cartel-style agenda control.

6 Nash equilibria

We must now determine whether or not the paradox can happen in equilibrium. Citizen-candidate models can have equilibria in which more than two citizens become candidates, but examining such equilibria is unnecessary. The purpose of this paper is to show how the candidate furthest from the median voter can win, and that result is trivially easy to generate with more than two candidates. Therefore, we will focus on equilibria with precisely two candidates.

The critical question, then, is whether or not Proposition 5 applies to two-candidate equilibria in the citizen-candidate model. There are four conditions for Proposition 5.

Condition 4 is trivial since a majority party agenda setter can easily put an item on the agenda if she wishes. Condition 1 is a convenient but not necessary condition for the *Proximity Paradox*, and is perfectly plausible anyway. The question of whether or not Proposition 5 can operate in equilibrium is the combination of Conditions 2 and 3. In order for negative agenda control to permit majority party extremists to defeat minority party centrists, the two candidates must be located on opposite sides of the median voter. This is a particularly important condition given that the location of the median voter is a Condorcet winner, as Proposition 4 demonstrated. So, are there two-candidate equilibria in which the two candidates are on opposite sides of the median voter's location? In fact, Proposition 6 demonstrates that *all* two-candidate Nash equilibria to the model meet that condition.

Proposition 6 *For all Nash equilibria in which precisely two citizens*, A *and* B, *become candidates*, *the following inequality must hold*: $i_A < i_{MV} < i_B$, *where* i_{MV} *is the median voter's ideal point*, *and A's and B's ideal points are* i_A *and* i_B *respectively.*

Proof Proposition 6 is substantively identical to a condition for all citizen-candidate models. A strategy profile in a citizen-candidate model is a Nash equilibrium only if each citizen who enters to become a candidate is playing optimally by entering (given the other citizens' strategies), and if each citizen who does *not* enter is playing an optimal strategy by *not* entering (given the other citizen's strategies). Consider a strategy profile in which precisely two citizens, A and B, become candidates. We will consider two additional citizens who are choosing not to become candidates: one citizen located at the median voter's ideal point i_{MV}, and one citizen, c, located slightly to the right of i_{MV}. We will demonstrate that this strategy profile is only a Nash equilibrium if $i_A < i_{MV} < i_B$, where i_{MV} is the median voter's ideal point.

Suppose, first, that $i_A = i_{MV}$. Either $i_B \neq i_{MV}$, or $i_B = i_{MV}$. Let us first consider the possibility that $i_B \neq i_{MV}$. By Proposition 4, Candidate A will deterministically beat Candidate B as long as there is at least one roll call vote on which they disagree. Therefore, Candidate B is playing a suboptimal strategy by entering the race. She is paying the cost of entry with no possibility of winning. If she abstained from becoming a candidate, she would receive the same policies, but not have to pay the cost of entering the race. Therefore, there are no two-candidate Nash equilibria in which precisely one citizen at the location of the median voter emerges to become a candidate because the candidate *not* at the location of the median voter would receive more utility by not entering.

Next, suppose $i_A = i_{MV}$ and $i_A = i_{MV}$, and nobody else enters. Consider citizen C's decision not to enter the race (C is the citizen slightly to the right of the median). C is playing a suboptimal strategy by not entering the race. If she entered, A and B would evenly split just over half the vote (slightly more than 25% each), and C would receive just under 50%, thus deterministically winning a plurality, and receiving the benefit, as well as her ideal roll call vote on every vote. Thus, the strategy profile in which only A and B enter such that $i_A = i_B = i_{MV}$ is not a Nash equilibrium because one of the citizens who is not entering is playing suboptimally. Since there are no two-candidate Nash equilibria with precisely one candidate at the location of the median voter, and no two-candidate Nash equilibria with both candidates at the location of the median voter, it follows that there are no two-candidate Nash equilibria with either candidate at the location of the median.

Furthermore, suppose A and B enter such that $i_A < i_{MV}$ and $i_B < i_{MV}$, and nobody else enters. Consider an arbitrary citizen located at i_{MV}. If she entered, she would win a majority of the vote, receive the benefit of victory, and her ideal vote on every roll call. Thus, she is playing a suboptimal strategy by not entering the race. Therefore, a strategy profile in which precisely two candidates emerge such that $i_A < i_{MV}$ and $i_B < i_{MV}$ is not a Nash equilibrium. Therefore, all two-candidate Nash equilibria are characterized by the following inequality: $i_A < i_{MV} < i_B$. Thus, the conditions for Proposition 5 apply to all two-candidate Nash equilibria. The key to these proofs is that a Nash equilibrium in a citizen-candidate model requires that each candidate's decision to enter the race be optimal given everyone else's strategies, and that each non-candidate's decision *not* to enter the race be optimal given everyone else's strategies.

The *Proximity Paradox* cannot explain extremists defeating moderates if either candidate is located at the median voter's ideal point because of Proposition 4. However, Proposition 6 shows that Proposition 4 will not apply to any two-candidate Nash equilibria because in all two-candidate equilibria, the candidates deviate from the median voter's ideal point in opposite directions. Hence, all two-candidate equilibria permit a *Proximity Paradox* for the median voter, and by Proposition 5, negative agenda control necessarily creates such a paradox.

Clearly, all two-candidate Nash equilibria require that each candidate has a 0.5 probability of winning, and Proposition 5 shows that the electorate can be indifferent between two candidates even when one candidate is closer to the median voter. Hence, negative agenda control creates two-candidate equilibria in which a candidate further from the median voter has a non-zero probability of defeating a candidate closer to the median in equilibrium because of an asymmetric set of roll call votes. Thus, the Proximity Paradox, which is caused by party agenda control in Congress, can explain why extremists defeat moderates, in equilibrium. □

7 Empirical predictions

The predictions of this model differ significantly from Downsian models, in which the candidate closest to the location of the median voter deterministically wins. Of course, this is not the first paper to predict violations of proximity voting. However, whereas previous models have generated those violations with valence dimensions, strategic voting, and so on, this model has a more parsimonious explanation—voters receive utility from the roll call votes implied by a winner's location rather than from the location itself. The only analytic trick here is acknowledging the substantive political process by which winning legislative candidates actually carry out their campaign platforms. However, parsimony is not the only contribution of this paper. The model presented here generates specific and unique predictions about *how* violations of proximity voting will occur. The electoral advantage that generates non-proximity voting here is asymmetric—it is held uniquely by candidates of the majority party because it is created by the majority party's use of agenda control. Thus, if proximity-based voting would give a majority party candidate X% of the vote, we should expect that candidate to actually receive $(X + a)\%$ of the vote, where a represents the advantage created by *not* having to cast electorally harmful roll call votes, combined with the electorally harmful roll call votes that a legislator from the opposing party would have to cast.

Moreover, the advantage should change hands *after* party control of the chamber changes because holding a majority is what gives a party the ability to protect its incumbents from having to cast electorally dangerous votes. Thus, if the Democrats are the majority party going into the election at time t, then Democratic candidates should have electoral advantages beyond what spatial theory would predict during the election at time t. However, if the Republicans manage to win a majority of seats at election t despite their disadvantage, *they* would possess the electoral advantage in the election at time $t + 1$, because they would suddenly have the ability to protect their own incumbents from dangerous votes, and Democrats would not. Thus, the electoral advantage creating violations of proximity voting should be asymmetric, and it should change hands *following* a change in control of the House.

These predictions are, of course, falsifiable, and they differ from the predictions of other models with non-proximity voting. For example, if voters have monotonically increasing preferences over valence characteristics, like competence and honesty, then extremists can defeat moderates by having advantages in competence or honesty. However, such effects should be indifferent to party. The legislative agenda effect is party-specific, and changes whenever control of the chamber changes.

However, valence traits like competence and honesty differ significantly from Stokes' (1963) original conception of valence. In that paper, Stokes distinguished between positional issues, represented by conventional spatial theory, and valence issues, on which voters agree on goals, but disagree on who can best achieve them. If one party has an advantage with respect to a valence issue, such as a Democratic advantage with education, or a Republican advantage with national security, then valence issues can create an electoral advantage beyond what spatial positions predict, and that advantage would be asymmetric. However, such advantages should either be static over time, or change *prior* to a change in control of the legislative chamber because a change in valence advantages would *cause* a change in control of the legislative chamber. However, the legislative agenda effect should not be static, and it should follow rather than precede a change in control of the chamber.

Voters might prefer an extremist to a moderate if they are trying to balance Congress and the presidency, as in Lacy and Paolino (1998), but if that is the case, the advantage should be held by the party opposed to the president, and when the same party controls Congress and the presidency, that is the minority party, not the majority party.

Along similar lines, voters might deviate from proximity voting in order to affect legislative policy output by shifting the chamber median. However, if they do so, we would expect the motivation to be shifting the chamber median towards the center. Naturally, that would imply favoring minority party candidates, whereas the legislative agenda prediction is a bias towards majority party candidates.

Partisan bias can also cause deviations from proximity voting, even among ideologues (Jessee 2009). However, independents are not subject to that bias. The independent voters should be equally susceptible to the legislative agenda effect, and that effect should be asymmetric, tracking majority party status in Congress over time. Hence, the agenda effect should be separable from the partisan effect. Of course, Jessee (2009) finds that independents behave in a way that tracks their spatial preferences very well, but that is with voters in presidential elections, and the legislative agenda effect should apply primarily to congressional elections. Presidents, after all, are less constrained by the legislative agenda in how they implement their platforms. They can also issue executive orders, apply budgetary discretion, and use a variety of other tools with fewer constraints than Members of Congress.

The critical point about this model, then, is not simply that it predicts violations of proximity voting, but that it predicts violations of proximity voting that favor the majority party, and that persistent advantage separates the predictions of this model from others with non-proximity voting.

Of course, nothing in this paper should be taken to imply that minority party incumbents cannot have advantages that allow them to beat more centrist candidates, such as advantages from valence characteristics like competence or honesty, but the advantage that comes from agenda control in Congress should belong uniquely to candidates of the majority party, and to the degree that we see a consistent advantage for majority party candidates above and beyond what other theories would predict, that suggests an electoral benefit that comes from agenda control.

8 Implications for general spatial theories

While the model presented here predicts a spatial advantage for majority party legislative candidates, that majority party advantage comes from the interaction between two factors—the structure of the legislative agenda under cartel theory, and the way that rational voters should respond to the legislative agenda. Hence, under a different model of the legislative agenda, voters motivated by the same reasoning used in this paper may behave very differently. The obvious next question, then, is as follows: if voters evaluate candidates on the basis of the bundles of roll call votes implied by their locations rather than the locations themselves, how would they behave under different assumptions about the structure of the legislative agenda? That is a critical question for future research.

The important point for spatial theory more broadly, then, is not that majority party candidates have an inherent electoral advantage, but that we must build models of elections that acknowledge the substantive political process by which winning legislative candidates implement their platforms. That process is not a single action. It is the incremental process of casting roll call votes, and moving to a roll call-based conception of platform implementation has important implications for spatial theory beyond the more narrow predictions of this model.

Similarly, it should be noted that the predictions of this model may also be affected by the use of a citizen-candidate model, in which candidates are constrained to run on their sincere ideal points rather than on strategically selected platforms that may be insincere. The implications of allowing voters to consider the structure of the legislative agenda when evaluating legislative candidates should also be considered in models that permit strategic positioning in the vein of either Downs (1957) or Wittman (1977, 1983).

Finally, it is important to consider the fact that the model in this paper assumed that voters evaluate candidates based on retrospective evaluation of past roll call votes rather than prospective evaluation of future roll call votes. The substantive process that produces the proximity paradox is identical, regardless of whether the evaluation process is prospective or retrospective, but Appendix C discusses the applications of these ideas to a prospective model.

9 Conclusions

If voters have preferences over the bundles of roll call votes implied by candidate locations rather than preferences over the locations themselves, then they can sincerely prefer distant candidates to more proximate candidates, even when they have single-peaked, symmetric preferences and perfect information. Furthermore, negative agenda control in the House of Representatives creates precisely the types of agendas in which this paradox occurs. Thus, party agenda control can allow majority party extremists to defeat more moderate minority party candidates. More broadly, though, this paper argues that spatial theory has ignored the substantive political process by which winning legislative candidates implement their platforms—the process of casting roll call votes. If voters consider what candidate locations imply about roll call votes, they will behave very differently than if they simply assume that the winner implements her platform in a single action. It is important for spatial theory to consider the process by which winning candidates implement their campaign platforms because they do not do so with a single action, as conventional spatial theory assumes.

Acknowledgements The author thanks the following individuals for their comments on this paper: Jim Adams, Stephen Ansolabehere, Karen Beckwith, Linda Fowler, Kelly McMann, Sam Merrill, Eliot Posner, and Joe White. An earlier version of this paper was presented at the 2007 Annual Meeting of the Midwest Political Science Association.

Appendix A: Limitations of the Proximity Paradox

There are several caveats to the possibility of the Proximity Paradox. First inequality (5) must be false if both candidates are located on the same side of the voter's ideal point.

Lemma 1 *If $i_A < i_B < i_V$, the voter will prefer B's bundle of roll call votes to A's bundle if there are any votes between l and c such that $i_A < (l + c)/2 < i_B$. Otherwise, the voter will be indifferent between A's bundle of votes and B's bundle of votes.*

Proof Suppose $i_A < i_B < i_V$. In order for the voter to prefer the bundle of votes offered by A to the bundle offered by B, there must be at least one roll call vote such that A votes consistently with the voter's preferences, but B does not. Such votes cannot exist. If B disagrees with the voter on a vote, that implies that $i_B < (l + c)/2 < i_V$. However, since $i_A < i_B$, it follows that $i_A < (l + c)/2 < i_V$ so if B disagrees with the voter on a roll call vote, A must also disagree. Thus, there can be no roll call votes on which the voter disagrees with B, but agrees with A. Thus, the voter cannot strictly prefer A's bundle of votes to B's bundle of votes. If there are any votes such that $i_A < (l + c)/2 < i_B$, A would have disagreed with the voter, but B would not, in which case the voter prefers B's bundle of votes. If there are no votes such that $i_A < (l + c)/2 < i_B$, then the voter is indifferent between A's bundle and B's bundle, but the voter cannot prefer A's bundle of votes to B's bundle of votes. □

Lemma 2 *A voter cannot prefer a candidate located anywhere other than her ideal point to a candidate located at her ideal point.*

Proof A candidate located at the voter's ideal point will cast all votes consistently with the voter's preferences. A candidate located anywhere else may cast votes inconsistently with the voter's preferences. Thus, if candidate A is located at $i_A < i_V$, and candidate B is located at $i_B = i_V$, the voter will prefer candidate B's bundle of votes to candidate A's bundle if there are any roll call votes such that $i_A < (l + c)/2 < i_B$, and the voter will otherwise be indifferent between their bundles of votes. □

Appendix B: Group preferences and median voter preferences

While Proposition 5 shows that the candidate closest to the median voter does not necessarily win, we might pose a weaker version of a general median voter theorem: does a group prefer the option preferred by the median voter, regardless of whether or not the median voter's preferred candidate is closest to her ideal point? In fact, we cannot even make that argument without additional restrictions on the forms of voters' utility functions, as we shall see below.

Suppose $i_A < i_M < i_B$ and $i_M - i_A < i_B - i_M$. Suppose also that there is a proximity paradox such that the median voter prefers B to A, even though A is closer to her ideal point. We can predict that B will win the election only if all voters to the right of the median also prefer B. If not, A might still win. Therefore, the group's preferences only reflect the median voter's preferences if proximity paradoxes are one-directional. Without very restrictive assumptions, they are not.

Proposition 7 *Suppose all voters have single-peaked, symmetric preferences. Suppose Voter* 1 *has an ideal point* i_1 *where* $i_A < i_1 < i_B$ *and* $i_1 - i_A < i_B - i_1$. *Suppose that there exists a proximity paradox such that Voter* 1 *prefers the bundle of roll call votes offered by B to the bundle of roll call votes offered by A. It does not follow that all voters with single-peaked, symmetric preferences and ideal points in the range* (i_1, i_B) *also prefer the bundle of votes offered by B.*

Corollary *If all voters have single-peaked, symmetric preferences, the candidate preferred by the median voter does not necessarily win.*

Proof Consider two alternative forms of single-peaked, symmetric utility functions: $U(p) = -(i-p)^2$, and $U(p) = -|i-p|^{1/2}$. Suppose Voter 1 has a utility function of $U_1(p) = -(2-p)^2$, and Voter 2 has a utility function of $U_1(p) = -|3-p|^{1/2}$. Now, suppose candidate A is located at 1, and candidate B is located at 4. Thus, Voter 1 is closer to A, and Voter 2 is closer to B. Proving Proposition 5 requires finding an agenda of roll call votes such that Voter 1 prefers the bundle of votes offered by Candidate B, but Voter 2 prefers the bundle of votes offered by Candidate A. Suppose the first vote is between $l_1 = 1$ and $c_1 = 2$, and the second vote is between $l_2 = 3$ and $c_2 = 3.4$.

$$U_1(A) = -(2-1)^2 - (2-3)^2 = -1 - 1 = -2$$
$$U_1(B) = -(2-2)^2 - (2-3.4)^2 = 0 - 1.4^2 = -1.96$$

Thus, Voter 1 prefers B because of a proximity paradox. Now, let us examine Voter 2's preferences between A and B *given the same roll call votes.*

$$U_2(A) = -|3-1|^{1/2} - |3-3|^{1/2} = -2^{1/2} = -1.414$$
$$U_2(B) = -|3-2|^{1/2} - |3-3.4|^{1/2} = -1 - 0.4^{1/2} = -1 - 0.632 = -1.632$$

Thus, Voter 2 prefers A because of another proximity paradox. So, the direction of a proximity paradox for any given set of roll call votes is not necessarily consistent. This single example is sufficient to prove Proposition 5, which merely argues that we cannot rule out paradoxes running in opposite directions with the same set of roll call votes. If $i_A < i_1 < i_2 < i_B$, it is possible for Voter 1 to prefer B and for Voter 2 to prefer A given the same set of roll call votes.

The corollary follows clearly from this point. Suppose that $i_A < i_M < i_B$ and $i_M - i_A < i_B - i_M$. Proposition 1 shows that the Median Voter might prefer Candidate B to Candidate A, despite the fact that A is closer to the Median Voter's ideal point. It follows that the electorate prefers B only if all voters to the right of the Median also prefer B. However, Proposition 5 shows that voters to the right of the median might prefer A. If all voters to the left of the median are also to the left of A, then all such voters prefer Candidate A by Lemma 1. By Proposition 5, at least one voter to the right of the median might also prefer A, and if that is the case, then A will win even though the median voter prefers B. Thus, the preferences of the group *do not* necessarily reflect the preferences of the median. □

Appendix C: Prospective candidate evaluations and endogenous agenda formation

This paper has generated violations of proximity voting based on retrospective analysis of past roll call votes, but the calculus of candidate evaluation would be identical with prospective evaluation. However, the possibility of prospective analysis raises two questions. First, how would voters know the future agenda? Secondly, what if the election result influences the future agenda? We must consider each issue in turn.

To a large degree, campaigns center around a conflict over the agenda anyway. One candidate may attempt put a minimum wage increase on the agenda. Doing so is an attempt to indicate to voters that there will be a roll call vote in which the status quo will be the preservation of the current minimum wage, and the alternative will be an increase of some specific amount. Voters might infer that the winner of the election will cast a roll call vote on such a proposal, and conclude that this roll call will be a component of the bundle of votes that each candidate offers. The opposing candidate might attempt to put a tax cut on the agenda. Doing so is an attempt to indicate to voters that there will be a roll call vote in which the status quo will be the current tax rate, and the alternative will be a decrease of some specific amount. Voters might infer that the winner of the election will cast a roll call vote on such a proposal, and conclude that this roll call will be a component of the bundle of votes that each candidate offers. Each candidate attempts to stack the future agenda with issues that will allow them to maximize more than half the voters' utilities *even if they are located further from the median than their opponents*. Drawing lines of cleavage, as Schattschneider (1960) would say, is not just what roll call votes do, it is what campaigns do. Thus, when we see opposing campaigns struggle to set the agenda, they are struggling to control the issues that voters think will come up once either candidate is in office. They are struggling to construct the bundle of roll call votes that voters associate with each candidate. In doing so, they are facilitating the proximity paradox by asking voters to use the same calculus they use in the model presented here. Thus, the reasoning for this model works equally well in a prospective context, although the mechanism producing the paradox may be the nature of the campaign rather than agenda control in Congress.

So, what if the future agenda depends on the results of the election? In order to answer that question, we must examine the means by which election results can affect the agenda. First, suppose that there is an open amendment rule. If any legislator can demand a floor vote on any proposal, then a candidate closer to the voter's ideal point cannot be stopped from offering a proposal to move policy towards that voter's ideal point. If that is the case, then the paradox may cease to operate, although making that determination will require further analysis based on different assumptions about the legislative agenda. Such analysis may be better suited to Senate elections than to House elections, though, given the importance of the Rules Committee in the House, and the absence of a comparable committee in the Senate.

An alternative way in which the agenda might be influenced by election results is if election results determine which party controls the chamber. If the winner of the election will give control to that candidate's party, then it may again be rational for the voter to choose the most proximate candidate because that candidate's party might impose roll call votes that will both make the voter prefer her bundle of votes, and provide more total utility. Ironically, though, the proximity paradox also demonstrates that the two are not synonymous.

Finally, the election result may indirectly affect the agenda by affecting the gridlock interval. Of course, in a chamber with 435 Members, one election will generally have a small effect on the location of the floor median, so the range of the gridlock interval will only be minimally affected by the winner of any one election.

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