

Diamagnetic persistent currents and spontaneous time-reversal symmetry breaking in mesoscopic structures

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Recently, new strongly interacting phases have been uncovered in mesoscopic systems with chaotic scattering at the boundaries. The new phases are the mesoscopic analogue of spontaneous distortions of the Fermi surface induced by interactions in bulk systems and can occur in any Fermi-liquid channel with angular momentum m . Here we show that the phase with m even has a diamagnetic persistent current (seen experimentally but mysterious theoretically), while that with m odd can be driven through a transition which spontaneously breaks time-reversal symmetry by increasing the coupling to dissipative leads. Our analysis is reliable when the dimensionless conductance of the system is large, is nonperturbative in both disorder and interactions, and applies to ballistic structures with strong interactions.

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The interplay of disorder and interactions is a rich source of unexplained phenomena in the bulk, especially in two dimensions,¹ despite three decades of theoretical effort.²⁻⁴ In mesoscopic systems one is confronted with phenomena not seen in the bulk, such as Coulomb blockade oscillations^{5,6} of the zero-bias conductance, or persistent currents in mesoscopic rings in a small external magnetic field⁷⁻¹⁴ (for reviews see Ref. 15). It has been realized in the last several years that one needs to take interactions seriously in order to understand the experimental Coulomb blockade peak spacing statistics.⁶ This understanding has led to universal Hamiltonian^{16,17} treatments for weak coupling (electron-gas parameter $r_s = 1/a_0\sqrt{\pi\rho} \approx 1$, where a_0 is the Bohr radius and ρ is the electron density), in which a constant charging interaction and a constant exchange interaction are kept in addition to the single-particle energy.

Recently, two of us investigated¹⁸ the stability of the universal Hamiltonian to other interactions of the Landau-Fermi-liquid type,¹⁹ which are expected to be present in ballistic quantum dots with chaotic boundary scattering, but are not in dots deep in the diffusive limit.²⁰ The Landau interaction $u[\theta(\mathbf{k}) - \theta(\mathbf{k}')]]$ depends only on the angle between the momenta \mathbf{k}, \mathbf{k}' of the interacting particles,¹⁹ and can be parametrized in two dimensions by a set of Landau parameters u_m

$$u(\theta - \theta') = u_0 + \sum_{m=1}^{\infty} u_m \cos m(\theta - \theta'). \quad (1)$$

The values of the Landau parameters depend on the strength and range of interactions, and can only be accessed numerically²¹ for large r_s (strong interactions). It was found¹⁸ that while the universal Hamiltonian was stable in the regime $u_m > u_m^* = -1/2 \ln 2$, it became unstable to a mesoscopic Pomeranchuk²² phase for $u_m < u_m^*$. Soon afterwards one of us and Shankar²³ showed how to construct the large- N theory of the strong-coupling regime (the dimensionless conductance g plays the role of N). Details of this treatment will appear soon.²⁴

In this paper, treating Coulomb blockade and persistent currents within the same approach, we show that the mesoscopic Pomeranchuk phases display unexpected signatures in the persistent current, including a diamagnetic persistent current (seen experimentally¹²⁻¹⁴ but so far unexplained) in a model without superconductivity for m even, and spontaneous time-reversal symmetry breaking for m odd.

We want the effective Hamiltonian in an energy window of width the Thouless energy E_T around the Fermi energy. This is the regime of validity¹⁵ of random matrix theory (RMT) (Ref. 25). For ballistic structures $E_T = \hbar v_F / L$, where v_F is the Fermi velocity and L is the linear system size. The dimensionless conductance is defined as the number of single-particle energy states (of mean spacing Δ) in this window $g = E_T / \Delta$. Our effective Hamiltonian^{18,23} has a noninteracting part representing the chaotic scattering at the walls, and a Fermi-liquid-like interacting part which conserves momentum. The order parameter²³ $\boldsymbol{\sigma}$ of the Pomeranchuk phase is a two-dimensional vector whose magnitude σ and direction χ represent the size and direction of the maximum Fermi-surface distortion. The shape of the deformed Fermi surface is given by $\sigma \cos(m\theta - \chi)$, where m is the angular momentum channel in which the instability occurs. To determine the behavior of $\boldsymbol{\sigma}$ the fermions are integrated out and an effective action is obtained,²³ the dominant part of which is self-averaging for large g .

$$\begin{aligned} \bar{S}_{eff} = g^2 \int dt & \left[\frac{|\boldsymbol{\sigma}|^2}{2\Delta} \left(\frac{1}{|u_m|} - \frac{1}{|u_m^*|} \right) + \lambda (\boldsymbol{\sigma}^2)^2 \right] \\ & + g \int \frac{d\omega}{2\pi} |\boldsymbol{\sigma}(\omega)|^2 f(\omega). \end{aligned} \quad (2)$$

The “kinetic” term $f(\omega)$ behaves like ω^2 for $\omega \ll \Delta$ and like $|\omega|$ for $\omega \gg \Delta$, indicating the Landau damping of $\boldsymbol{\sigma}$. For large g the saddle point of this action dominates,²³ since fluctuations are down by $1/g$. A strong enough attractive Landau parameter $u_m \leq u_m^*$ leads to symmetry breaking.^{18,23}

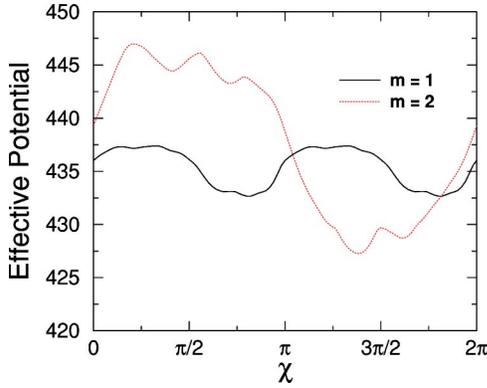


FIG. 1. (Color online) The effective potential in the Mexican hat in units of Δ for a particular realization plotted as a function of the angle χ for the cases m even and odd for $g=20$ and $u_m = -1.7$.

We have numerically evaluated the effective potential for the collective variable σ and analyzed its dependence on external magnetic flux. This effective potential can be obtained as the ground-state energy $\mathcal{E}(\sigma)$ of a noninteracting fermion Hamiltonian, where $\sigma = \sigma(\cos \chi \hat{i} + \sin \chi \hat{j})$ appears as a parameter:

$$[H_\sigma(\chi)]_{\alpha\beta} = \varepsilon_\alpha \delta_{\alpha\beta} - g \sigma M_{\alpha\beta}(\chi), \quad (3)$$

where the first term encodes the chaotic scattering (with the eigenvalues ε_α controlled by RMT), and $M_{\alpha\beta}(\chi) = \sum_{\mathbf{k}} \psi_\alpha^*(\mathbf{k}) \psi_\beta(\mathbf{k}) \cos(m\theta_{\mathbf{k}} - \chi)$ represents the coupling between the collective mode σ and particle-hole excitations of the fermions.²³ Here the $\psi_\alpha(\mathbf{k})$ are the noninteracting chaotic eigenfunctions in the dot written in the basis of g approximate momentum states \mathbf{k} , which are assumed to lie on the Fermi circle making an angle $\theta_{\mathbf{k}}$ with an arbitrarily chosen x direction (for details see Refs. 23,24). In the large- g limit, the ground-state energy of the system in the strong-coupling regime is²³ simply the value of the effective potential at its global minimum. This energy automatically contains the contributions which are specific to the particular disorder realization in addition to the self-averaging contributions determined earlier.²³ The averaged energy landscape of σ in the strong-coupling phase would be a ‘‘Mexican hat,’’ with rotational symmetry.²³ The sample-specific contributions break this symmetry completely for m even, leading to a single minimum, and to a two-fold symmetry for m odd, as can be seen in Fig. 1.

To see the relation of the above to the persistent current we note that¹⁵

$$I(\phi) = - \frac{\partial F(\phi)}{\partial \phi}, \quad (4)$$

where ϕ is the external flux piercing the sample. At zero temperature the free energy F is just the ground-state energy \mathcal{E} , so we desire to obtain \mathcal{E} as a function of ϕ . We obtain this by taking the noninteracting part of the Hamiltonian from the RMT ensemble of crossover Hamiltonians²⁵ parametrized as

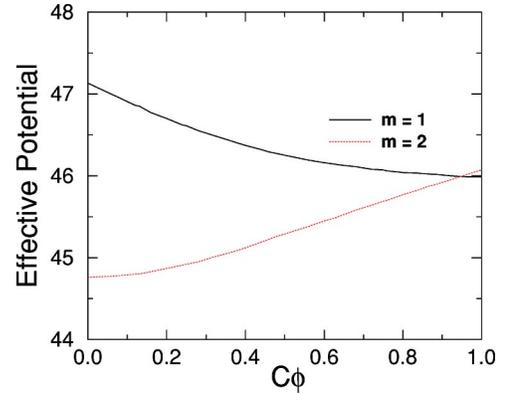


FIG. 2. (Color online) The ensemble-averaged minimum of the effective potential in units of Δ (for $g=20$ and $u_m = -1.7$) as a function of the crossover parameter ϕ for $m=1$ and $m=2$.

$$H_{cross} = \sqrt{\frac{1}{1 + C^2 \phi^2}} [H_S + C \phi H_A], \quad (5)$$

where $H_{S,A}$ represent symmetric (\mathcal{T} preserving) and antisymmetric (\mathcal{T} breaking) random matrices drawn from their respective normalized ensembles,²⁵ and C is a factor of order unity which depends on the shape of the sample and the precise nature of the chaotic scattering at the boundary.¹⁷ The flux dependence of the interacting part is suppressed by powers of g , as we will elucidate in a future publication.²⁶

Figure 2 shows the dependence of \mathcal{E} on the crossover parameter $C\phi$ for m even and odd, and clearly shows the diamagnetic behavior for m even. To see why this is special, consider what is known about persistent currents.¹⁵ In a mesoscopic ring penetrated by a flux, the ground-state energy has to be periodic in the flux, since an integer number of flux quanta can be gauged away.

$$I_{pers}(\phi) = - \frac{\partial F}{\partial \phi} = I_1 \sin(2\pi\phi/\phi_0) + I_2 \sin(4\pi\phi/\phi_0) + \dots, \quad (6)$$

where $\phi_0 = h/e$ is the flux quantum. Only the even moments I_{2n} survive disorder averaging.¹⁵

In the noninteracting case the typical fluctuating values of the Fourier coefficients are (for small n) $I_{n,typ} \approx E_T/\phi_0$ while the average is $\langle I_{2n} \rangle \approx \Delta/\phi_0$. Experiments typically measure $\langle I_{2n} \rangle$ and a few other low-order harmonics.⁹⁻¹⁴

Interactions, when included in renormalized first-order perturbation theory,⁸ produce $\langle I_2 \rangle \approx \mu^*(E_T/\phi_0)$ where μ^* (of order 1) is the dimensionless Cooper-channel interaction at low energies. Thus, interactions enhance the average persistent current, but if $\mu^* > 0$ $\langle I_2 \rangle$ should be paramagnetic, while if $\mu^* < 0$ it should be diamagnetic. This prediction,⁸ while of the same order of magnitude as the experiments,⁹⁻¹⁴ has the wrong sign. Materials that show no sign of superconductivity (implying that $\mu^* > 0$) show¹¹⁻¹⁴ a diamagnetic $\langle I_2 \rangle$. Many explanations have been proposed to account for this puzzle (a recent one being Ref. 27), but the question remains open, as summarized in Ref. 28. In this context a diamagnetic persistent current of order E_T/ϕ_0 in a model without superconductivity is striking. The fact that our treatment is nonperturbative in the interactions²³ enables us to evade the usual sign.⁸ Our approach, while suggestive, is not

directly applicable to the experiments on Au and Ag rings^{12,14} since the samples are not likely to be in the strong-coupling regime, and are not fully in the ballistic limit (the elastic mean-free path is of the same order as the system size). On the other hand, our theory would apply directly to ensembles of ballistic GaAs rings of the type used in Refs. 11,13, but at stronger coupling.

Let us now turn to m odd. The exact degeneracy of the two global minima separated by π in the angle χ can be seen from Fig. 1, and can be proved analytically using the relation $H_\sigma^*(\chi) = H_\sigma(\chi + \pi)$. The two degenerate minima are related by the time-reversal transformation \mathcal{T} . A particular value of χ leads to a distortion of the Fermi surface along the direction specified by χ . Under \mathcal{T} , $\mathbf{k} \rightarrow -\mathbf{k}$ and a distortion of the Fermi surface for odd m maps to an inequivalent state at $\chi + \pi$ with the same energy, since the underlying Hamiltonian is \mathcal{T} invariant. The ground state of a Hamiltonian quantum system with a twofold degenerate potential is the symmetric combination of the two minima. This applies to the isolated mesoscopic structure, whose dynamics is Hamiltonian at energy scales smaller than Δ [for energy scales in the range $\Delta \ll \omega \ll E_T$ the dynamics is dissipative with ohmic dissipation, see Eq. (2)]. The splitting between the symmetric and antisymmetric combinations is the tunneling amplitude between the two minima, here Δe^{-g} . The two minima correspond to states carrying opposite persistent currents, and are macroscopically distinguishable.

The coupling of the mesoscopic structure to the leads produces ohmic dissipation at arbitrarily low energies. This is precisely the case of the Caldeira-Leggett model²⁹ considered and solved by Chakravarty,³⁰ and Bray and Moore.³⁰ The effective action of our model at low energies ($\omega \ll \Delta$) is

$$g \int dt \left(V[\chi(t)] + \frac{1}{2} (d\chi/dt)^2 \right) + \frac{2g\Gamma^2 \langle \sigma^2 \rangle}{\pi \Delta^2} \int \frac{dt dt'}{(t-t')^2} \sin^2 \left(\frac{\chi - \chi'}{2} \right), \quad (7)$$

where χ is the angle of σ in the Mexican hat, $V(\chi)$ is the doubly degenerate realization-specific potential of Fig. 1, and Γ is the level width induced by coupling to the leads. The long-range interaction in imaginary time comes from a $|\omega|$ kinetic term, which in turn arises from the Landau damping of σ due to decay into particle-hole pairs at arbitrarily low energies, possible because each formerly sharp level α is broadened by coupling to the leads.

The model has a weak-dissipation phase in which the ground state is still the symmetric superposition of the two minima, and a strong-dissipation phase in which the particle is localized in one minimum. The transition between the two phases³⁰ occurs for $g(\Gamma/\Delta)^2 \langle \sigma^2 \rangle \approx 1$. For large enough g even a weak coupling to the leads ($\Gamma \approx \Delta/\sqrt{g}$, since $\langle \sigma^2 \rangle \approx 1$) is sufficient to meet this criterion, and leads to localization in one minimum of the twofold degenerate effective potential even at zero temperature, corresponding to a spontaneous breaking of \mathcal{T} . The \mathcal{T} -breaking transition could be monitored by measuring the peak-height statistics.³¹

If one turns on an external flux, one minimum moves up

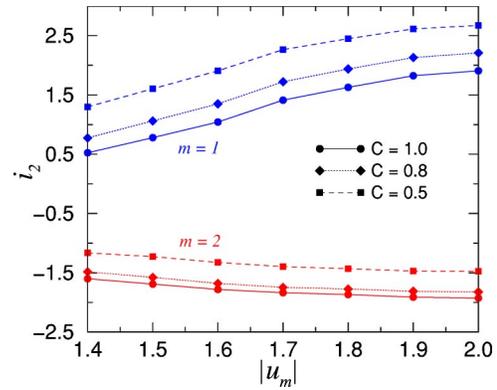


FIG. 3. (Color online) The ensemble-averaged second Fourier coefficient of the persistent current in units of Δ/ϕ_0 as a function of coupling $|u_m| \geq |u_m^*| = 1/\ln 2$ for $m=1$ and $m=2$, and $g=20$.

in energy as the flux increases while the other moves down. The ground state (which moves down) displays a paramagnetic persistent current of order E_T/ϕ_0 . In the isolated dot, or in the case with weak dissipation, one starts with a symmetric superposition of the two minima as the zero-flux ground state. As ϕ increases the system crosses over to fully \mathcal{T} -broken dynamics when the energy difference of the two minima is greater than their splitting in zero field, which is Δe^{-g} . Thus, the crossover will occur for an external flux $\phi_\chi \approx \phi_0 e^{-g}$, as compared to the noninteracting crossover flux $\phi \approx \phi_0/\sqrt{g}$. For strong enough dissipation, the ground state already breaks \mathcal{T} , and the variation of \mathcal{E} contains a term first order in ϕ , which implies a spontaneous persistent current at zero flux.

Figure 3 shows the ensemble-averaged second Fourier coefficient $\langle I_2 \rangle$ for m even and odd. In a microscopic model one would find a periodic behavior of \mathcal{E} with ϕ with period ϕ_0 . This cannot be captured by the crossover Hamiltonian [Eq. (5)], which is designed for use at small values of the crossover parameter. To circumvent this, we extend our numerical results obtained in the region $-\frac{1}{2} \leq C\phi/\phi_0 \leq \frac{1}{2}$ to other ϕ by assuming periodicity. Since we do not know the number C connecting the crossover parameter to the flux ϕ , there is an inherent ambiguity in this procedure. As can be seen in Fig. 3, the qualitative results are unaffected by the choice of C .

In summary, we have shown that some surprising signatures of the mesoscopic Pomeranchuk regimes show up in the persistent current. There is a diamagnetic persistent current (for m even) without any superconductivity. The m odd case undergoes a spontaneous time-reversal symmetry-breaking transition as the coupling to the leads is increased, and displays a spontaneous persistent current (at zero flux) in the \mathcal{T} -broken phase. It would be very interesting to explore the behavior of persistent currents in the quantum critical regime,³² where large fluctuations of the order parameter σ and finite quasiparticle lifetime³³ at low energies are expected.²³

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