Quantum Transport and the Electronic Aharonov-Casher Effect

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We show that the effects of spin-orbit interaction in disordered conductors are manifestations of the Aharonov-Casher effect in the same sense as the effects of weak magnetic fields are manifestations of the Aharonov-Bohm effect. We propose that, in semiconducting samples, conductance oscillations should occur due to the Aharonov-Casher effect, whose observation would constitute a thousandfold improvement in its measurement. As in the disordered Aharonov-Bohm effect, these oscillations have half the period expected from the conventional Aharonov-Casher effect.

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Aharonov and Bohm showed long ago that a magnetic field enters quantum mechanics in two distinct ways, a distinction most easily described in the semiclassical limit. In this limit, first the magnetic field determines the classical trajectory of the particle through the Lorentz force law (the dynamical effect), and second it contributes to the phase accumulated along the trajectory through the line integral of the vector potential along it (the geometric effect). The latter effect has no classical analog. The term Aharonov-Bohm (AB) effect is now commonly used even in situations for which the dynamical effect is not rigorously zero, but is negligible compared to the geometric effect.

In this sense there are now many well-known manifestations of the AB effect in the low-temperature transport properties of disordered normal conductors, e.g., weak-localization (WL) magnetoresistance [1], and the closely related Altshuler-Aronov-Spivak (AAS) effect [2], universal magnetoconductance fluctuations of mesoscopic samples [3], and persistent currents (AB magnetization) of arrays of rings [4]. In all of these phenomena weak magnetic fields (~ 100 G) are observed to have a significant effect at low T even though $\omega_c \tau \ll 1$ (ω_c is the cyclotron frequency and τ is the elastic scattering time), indicating that the dynamical role of the magnetic field is negligible compared to the impurity scattering. Low temperature is necessary so that the phase of the electron (which is smeared by inelastic scattering) remains well defined over large distances (typically μm). As in the ideal AB effect, periodic oscillations with enclosed magnetic flux are observed in multiply connected geometries, however, with an important complication unique to the disordered AB effect. Measurements on single rings show the fundamental hc/e periodicity expected, whereas measurements on arrays of rings or cylinders show a shorter period $hc/2e$ which is now understood to arise due to the effective disorder averaging which occurs when measuring many units at once [5]. Thus there is an effective charge doubling ($e \rightarrow 2e$) which is a special feature of the disorder-averaged response to a vector potential. Recently Aharonov and Casher (AC) [6] have noted an analog (more precisely, an electromagnetic dual) of the AB effect in the influence of electric fields on neutral magnetic moments through the spin-orbit (s.o.) interaction. Tests of this idea using neutron interferometry have been limited by the fact that for realizable electric fields and neutron fluxes, the phase shift is of the order of milliradians [7]. Below we shall extend the ideas of AC to disordered electronic systems; in this case the s.o. interaction can be interpreted as a non-Abelian vector potential coupled to the electronic spin, which upon disorder averaging leads to the appearance of integer instead of half-integer spin.

The clue to the relevance of the AC effect to disordered conductors is the well-known sensitivity of the disordered transport phenomena cited above to the s.o. interaction. For example, in the presence of strong s.o. scattering the weak localization effect is reversed in sign (antilocalization) as shown, e.g., in the classic experimental studies of metal films by Bergmann [1]. In metallic systems typically the s.o. scattering is due to the same impurities which cause the direct potential scattering; however, later studies have been done in semiconductors without inversion symmetry (such as GaAs) where it is believed that the dominant source of s.o. scattering is not the remote impurities but the nonrandom dipolar crystal field [8,9]. We shall see that this type of system presents the closest electronic analog of the AC effect. Theoretical works on s.o. scattering in disordered systems due to Chakravarty and Schmid [10] and Meir, Gefen, and Entin-Wohlman [11] have also emphasized in different ways that the importance of s.o. interaction in these systems arises from its coupling to the phase of the spin wave function and our work extends some of their ideas.

In this Letter we show that the s.o. sensitivity of disordered transport phenomena is a manifestation of the AC effect in exactly the same sense as the magnetic field sensitivity is a manifestation of the AB effect. Moreover, in contrast to the neutron interferometry experiments cited above, electric fields (due to either impurities or the crystal lattice) may exist inside solids that are sufficient to cause phase shifts of order unity. A new key notion is to treat these electric fields as fixed and obtain results for physical quantities as a function of the electric fields; this formalism allows us to both recover the classic results for random s.o. scattering as well as understand quantitatively the newer experiments in polar semiconductors for which the conventional theory of WL is inapplicable [8,9]. Because the large AC phase shift in polar semiconductors is tunable under certain conditions, it may be possible to see for the first time AC *oscillations* in the conductance of multiply connected semiconductor samples. Such experiments, if realizable, would constitute a thousandfold improvement over the previous observation of the AC effect.

For orientation, we compare the levels of a spinless charged free particle moving on a circle threaded by magnetic flux (AB effect) with the levels for a particle with magnetic moment μ on a circle threaded by a line of charge (AC effect). In the former case, the action for a classical orbit contains the (AB) contribution $2\pi\hbar\phi$, where ϕ is the flux in units of hc/e. It is easy to see, e.g., using the Bohr-Sommerfeld quantization condition, that the levels of the system are given by $(\hbar^2/2mr^2)(n-\phi)^2$, where n is an integer and r is the radius of the circle. In the latter case we suppose the dipoles are polarized parallel to the line of charge: This supposition permits the use of the Lagrangian derived by AC [6],

$$L=\tfrac{1}{2}mv^2-\boldsymbol{\mu}\cdot\mathbf{v}\times\mathbf{E}\,, \tag{1}$$

and it is easy to see that the levels are given by $(\hbar^2/2mr^2)(n\mp 2\lambda\mu/\hbar c)^2$, the sign depending on the polarization. In Eq. (1) and in all equations below we set $\hbar=c=1$. Here λ denotes the charge per unit length of the line of charge. Thus the levels of the system with spin $\mp$ in the presence of this electric field are the same as those of a spinless system with flux $\mp 2\lambda\mu/\hbar c$. Although we have derived this result semiclassically, and for a special electric field in the ring, Meir, Gefen, and Entin-Wohlman [11] have recently shown quite generally that the levels for a particle on a one-dimensional ring with arbitrary potential and s.o. scattering are the same as those of the spinless system at flux $\pm\delta$, where δ is determined by the electric field giving rise to the s.o. interaction [12]. In our present framework, this emerges as a consequence of the fact that in strictly one dimension the effect of s.o. interaction, like that of magnetic fields, has to be purely geometric.

To estimate the electric field needed to give an AC phase shift of order unity in electronic systems we use our simple model in which $\phi_{\rm AC}\sim 2\lambda\mu/\hbar c$, $E\sim\lambda/r$. We should take $r\sim l_{\rm in}$, $\mu=g^*\mu_{\rm Bohr}$; this yields

$$E\sim\frac{1}{g^*\alpha^2}\frac{a_0}{l_{\rm in}}\frac{e}{a_0^2}\,, \tag{2}$$

where a_0 is the Bohr radius and $\alpha\approx\frac{1}{137}$ is the fine-structure constant. Taking $l_{\rm in}\sim 1$ μm$\sim 10^4a_0$ gives $E\sim e/g^*a_0^2$; i.e., atomic strength electric fields are needed in metals, but weaker fields are sufficient in semiconductors where the g factor can be enhanced by 2 orders of magnitude. Since the impurity fields in metals *are* typically large enough to cause an AC phase shift of order unity, it is plausible that s.o. interaction can have a substantial effect on low-temperature interference phenomena in these systems as a manifestation of the AC effect.

To make this connection explicit we consider the Hamiltonian of an electron in a disordered solid within the effective-mass approximation:

$$\left[-\frac{1}{2m^*}\left[\boldsymbol{\nabla}-i\frac{\mu}{2}\boldsymbol{\sigma}\times\mathbf{E}(\mathbf{r})\right]^2+V(\mathbf{r})\right]\Psi(\mathbf{r},t)=-i\frac{\partial}{\partial t}\Psi(\mathbf{r},t)\,. \tag{3}$$

The electric field enters in the kinetic energy due to s.o. interaction and represents both the random electric field due to impurities and the slowly varying part of the crystal field (which would vanish in systems with inversion symmetry). This equation is equivalent to the familiar Pauli form of the Schrödinger equation with s.o. effects up to second order in v/c. It may be viewed as a description of a particle coupled to an SU(2) gauge potential $\mathbf{A}^{\rm s.o.}(\mathbf{r})=\frac{1}{2}\mu\boldsymbol{\sigma}\times\mathbf{E}(\mathbf{r})$, with spin playing the role of isospin, as noted by Goldhaber [13].

Equation (3) describes motion with a given random potential $V(r)$. In order to obtain statistical properties of the system one must average various physical quantities which are functionals of $V(r)$ over its different realizations. This leads to the well-known impurity-averaged perturbation theory [14], in which quantum interference effects arise from the statistical correlation of two single-particle Green functions. The important contributions come from the *Cooperon* (particle-particle ladder) and *diffuson* (particle-hole ladder) diagrams. Although the AC effect does not cancel in the diffuson (as does the AB effect) it requires a trivial modification of our treatment of the Cooperon, which will be our present focus. The Cooperon has been previously treated in a real-space approach by Altshuler *et al.* [15] in order to derive the $hc/2e$ AB effect and WL magnetoresistance and we proceed in close analogy to this work.

Reference [15] included the geometric effects of magnetic field on the average Green function, $\langle G(\mathbf{r},\mathbf{r}',\varepsilon_f)\rangle\sim\exp(|\mathbf{r}-\mathbf{r}'|/2l)$ (l is the elastic mean free path, and we neglect spin initially), within the semiclassical approximation simply by multiplying by a phase factor accumulated along the unperturbed classical path, as discussed above. For $|\mathbf{r}-\mathbf{r}'|\ll l$ this is just a straight line from $\mathbf{r}$ to $\mathbf{r}'$, hence

$$\langle G^{\pm}(\mathbf{r},\mathbf{r}',\varepsilon_f;B)\rangle=\exp\{\mp ie(\mathbf{r}-\mathbf{r}')\cdot\mathbf{A}(\tfrac{1}{2}(\mathbf{r}+\mathbf{r}'))\}\times\langle G^{\pm}(\mathbf{r},\mathbf{r}',\varepsilon_f)\rangle\,.$$

We have emphasized above that the s.o. interaction may be treated as a non-Abelian vector potential, and it is possible to show [10,16] that when the s.o. interaction has negligible dynamic effect a non-Abelian generalization of the above relation holds: The entire s.o. dependence in the Green function is contained in a unitary matrix (which is the analog of the phase accumulated along the classical trajectory) multiplying the Green function for the same system without s.o. scattering:

$$\langle G^{\rm s.o.\pm}_{\alpha\beta}(\mathbf{r},\mathbf{r}',\varepsilon_f)\rangle \simeq [\exp\{-\tfrac{1}{2}i\mu(\mathbf{r}-\mathbf{r}')\cdot\boldsymbol{\sigma}\times\mathbf{E}(\tfrac{1}{2}(\mathbf{r}+\mathbf{r}'))\}]_{\alpha\beta}\langle G^{\pm}(\mathbf{r},\mathbf{r}',\varepsilon_f)\rangle , \quad (4)$$

where $\langle G^{\pm}(\mathbf{r},\mathbf{r}',\varepsilon_f)\rangle$ denotes the Green function for a spinless particle and $\langle G^{\rm s.o.\pm}_{\alpha\beta}(\mathbf{r},\mathbf{r}',\varepsilon_f)\rangle$ for the full problem with spin and s.o. interaction. The 2×2 unitary matrix represents a spin rotation induced by the s.o. interaction; the particular rotation depends on the field along the classical path.

Physical observables such as linear response coefficients involve the average of the product of two Green functions; this average is not factorizable because the two Green functions are correlated by the underlying random potential [14]. In the semiclassical language one such correlation arises from interference of classical paths related by time-reversal symmetry [1,10]. This special correlation (the "Cooperon") satisfies the integral equation

$$C^{\rm s.o.}_{\alpha\beta\gamma\delta}(\mathbf{r},\mathbf{r}') = C^{\rm s.o.(0)}_{\alpha\beta\gamma\delta}(\mathbf{r},\mathbf{r}') + \int d\mathbf{r}''\, C^{\rm s.o.(0)}_{\alpha\mu\gamma\nu}(\mathbf{r},\mathbf{r}'')C^{\rm s.o.}_{\mu\beta\nu\delta}(\mathbf{r}'',\mathbf{r}') . \quad (5)$$

Here $C^{\rm s.o.(0)}_{\alpha\beta\gamma\delta} \equiv (v_f/\tau)\langle G^{\rm s.o.+}_{\alpha\beta}(\mathbf{r},\mathbf{r}')\rangle\langle G^{\rm s.o.-}_{\gamma\delta}(\mathbf{r},\mathbf{r}')\rangle$ and it can be expressed in terms of the spinless Green function using Eq. (4). Equation (5) may be formally solved in terms of the eigenfunctions of the $C^{\rm s.o.(0)}_{\alpha\beta\gamma\delta}$ in the usual fashion:

$$\sum_{\mu\nu}\int d\mathbf{r}'\, C^{\rm s.o.(0)}_{\alpha\mu\gamma\nu}(\mathbf{r},\mathbf{r}')\chi_{\mu\nu}(\mathbf{r}') = \lambda\chi_{\alpha\gamma}(\mathbf{r}) ,$$
$$C^{\rm s.o.}_{\alpha\beta\gamma\delta}(\mathbf{r},\mathbf{r}') = \sum_{\lambda}\left(\frac{\lambda}{1-\lambda}\right)\chi^{*}_{\beta\delta}(\mathbf{r}')\chi_{\alpha\gamma}(\mathbf{r}) . \quad (6)$$

The factor $\langle G^{+}(\mathbf{r},\mathbf{r}',\varepsilon_f)\rangle\langle G^{-}(\mathbf{r},\mathbf{r}',\varepsilon_f)\rangle$ in the integrand of the eigenvalue problem in Eq. (6) is sharply peaked about $\mathbf{r}\simeq\mathbf{r}'$; hence we Taylor expand the $\mathbf{r}'$ dependence of all other terms about $\mathbf{r}'=\mathbf{r}$ up to second order. Since the outer product of two spin rotations appears in $C^{(0)}$, it is natural to treat χ as a four-component spinor in the tensor product space of two spin-$\frac{1}{2}$ particles. On performing the $\mathbf{r}'$ integral we obtain an approximate differential form of the eigenvalue problem:

$$-D\tau[\nabla - \tfrac{1}{2}i\mu(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\times\mathbf{E}(\mathbf{r})]^2\chi(\mathbf{r}) = (1-\lambda)\chi(\mathbf{r}) , \quad (7)$$

where $\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2=\boldsymbol{\sigma}$ is the *total spin* operator for two spin-$\frac{1}{2}$ particles. This diffusion-type equation is our central result; it is more general than previous results because it is derived for a fixed electric field $\mathbf{E}(\mathbf{r})$ [17]. The formal similarity of Eq. (7) to the Schrödinger equation (3) brings out clearly its relationship to the AC effect. We consider its solution under various circumstances in order to gain insight into its physical content.

If heavy impurities are the dominant source of s.o. scattering, we must take the electric field in Eq. (7) as random [e.g., with zero mean and variance $(\mu/2)^2\langle E_iE_j\rangle \equiv L_{\rm s.o.}^{-2}\delta_{ij}$]; then we obain the standard result for the Cooperon [1,16], which leads to the well known phenomena such as antilocalization mentioned above.

We next consider Eq. (7) for the case of nonrandom electric fields as expected in semiconductor systems such as GaAs. First consider a thin film with a uniform electric field perpendicular to it. The uniform electric field represents the crystal electric field in this model. The solutions of Eq. (7) are readily obtained for this case. Using these solutions we obtain an antilocalization correction to the conductance in the strong s.o. scattering limit, which turns out to be similar in form [16] to that obtained for the random s.o. scattering case. One significant difference, however, is that the s.o. scattering length $L_{\rm s.o.}$ is now given by $\hbar c/g\mu E$, where E is the magnitude of the uniform electric field, and is *independent of the impurity concentration*, in contrast to the case with random s.o. scattering. This behavior of $L_{\rm s.o.}$ has been clearly observed in recent experiments of Dresselhaus *et al.* [9].

Finally we consider a narrow but multichannel (quasi-1D) ring penetrated by a flux ϕ in units of hc/e. The normal derivative of $C(\mathbf{r},\mathbf{r}')$ must vanish at the surfaces of the ring due to the vanishing of normal current [3]; thus to a good approximation the lowest eigenvalue solutions of Eq. (7) will be constant in the transverse directions and we seek solutions of the form $\chi(x)=\exp(iQx)\times U(x)\chi_0$, where $x=r\theta$ is the longitudinal coordinate along the ring, $U(x)$ is an operator in spin space, and χ_0 is a fixed spinor. $\chi(x)$ satisfies the flux-modified boundary conditions $\chi(x=L)=\exp(-i2\pi\phi)\chi(x=0)$. If we choose $U(x)$ such that

$$\left[\frac{\partial}{\partial x} - i\frac{\mu}{2}(\boldsymbol{\sigma}_1+\boldsymbol{\sigma}_2)\times\overline{\mathbf{E}(x)}\right]U(x)=0 , \quad (8)$$

where $\overline{\mathbf{E}(x)}$ denotes the cross-section averaged electric field and $U(x=0)=1$, we obtain the eigenvalue $1-\lambda = D\tau Q^2$, with the longitudinal boundary condition still to be applied. Equation (8) implies that $U(x)$ is a field-dependent rotation in the SU(2)×SU(2) spin space [in the language of gauge theory, $U(x)$ is the holonomy]; hence, the four eigenvalues of $U(x=L)$ are $\exp(\pm i2\pi\delta)$, 1 (twice). Choosing χ_0 to be an eigenvector of $U(x=L)$, it follows that Q is of the form $2\pi(n-\phi\pm\delta)/L, 2\pi(n-\phi)/L$.

It follows that any quantity that *depends only on the eigenvalues of the Cooperon* will satisfy

$$F^{\rm s.o.}(\phi) = F(\phi+\delta) + F(\phi-\delta) + 2F(\phi) , \quad (9)$$

where $F^{\rm s.o.}(\phi)$ denotes the value of that quantity with s.o. interaction at flux ϕ, and $F(\phi)$ denotes the same quantity for a spinless system at flux ϕ. Equation (9) has previously been derived for the specific case of the persistent current elsewhere [12]; here we generalize it to an arbitrary quantity of this type. The weak localization magnetoresistance, on the other hand, depends on both the eigenvalues and the eigenfunctions of Eq. (7). An explicit calculation based on Eqs. (7) and (8) shows that it satisfies [16]

$$\delta G^{\mathrm{s.o.}}(\phi)=\delta G(\phi+\delta)+\delta G(\phi-\delta)\,, \tag{10}$$

where $\delta G^{\mathrm{s.o.}}$ denotes the magnetoresistance with s.o. interaction at flux ϕ, and $\delta G(\phi)$ denotes the magnetoresistance for a spinless system at flux ϕ. A similar result was derived by Meir, Gefen, and Entin-Wohlman [11] for a strictly 1D system. Following their method, we average over δ appropriately for the case of strong, random s.o. scattering and obtain the sign reversal and a factor of $\frac{1}{2}$ reduction.

For III-V semiconductors such as GaAs the lack of inversion symmetry gives rise to large periodic dipolar crystal fields, and the AC phase shift δ will not be random, but will depend on the direction of motion. Recent experiments [9] which measured weak antilocalization in the two-dimensional electron gas in a GaAs heterostructure convincingly demonstrated that these crystal fields were the dominant source of s.o. scattering in such systems. They also showed by varying the electron density using a gate that $L_{\mathrm{s.o.}}$ was a strong function of carrier density; we interpret this dependence as arising from the strong dependence of the electron effective g factor on density. Thus these systems appear to meet the two requirements for measuring an oscillatory AC effect: (1) There are electric fields capable of producing AC phase shifts of order unity. (2) These phase shifts are tunable by an external parameter (gate voltage) over several periods. We propose then an experiment on multiply connected semiconducting samples similar to those studied in Ref. [9], which may exhibit conductance oscillations as a function of carrier density that would be an expression of the AC effect just as the Altshuler-Aronov-Spivak oscillations are an expression of the AB effect. One realization might be a GaAs sample fabricated with the geometry of Fig. 1, where the two branches are oriented such that the crystal field is zero for motion along the bent branch and nonzero for motion along the straight branch (see caption). The spin rotation δ determined by Eq. (8) should scale as $\mu EL/\hbar c$ (E is the electric field in the straight branch and L its length). δ may be tuned by varying the gate voltage leading to an oscillatory dependence [cf. Eq. (10)] of the conductance on gate voltage. In arrays of such elements the periodicity will be appropriate to the integer spin AC effect (just as arrays show the $hc/2e$ AB effect); whereas in single samples we expect that the spin-$\frac{1}{2}$ periodicity will be observable due to fluctuation effects, just as one sees the hc/e AB effect in single rings. Detailed theoretical study of various geometries and realistic discussion of possible experimental arrangements will be given elsewhere [16].

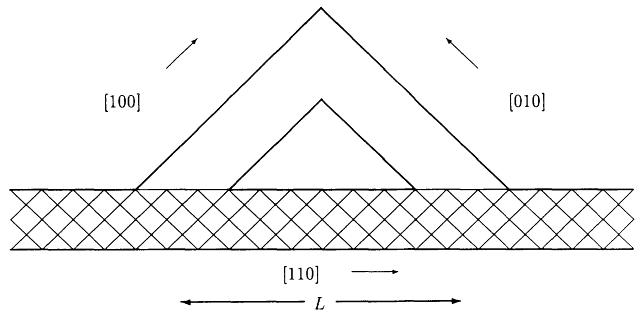

FIG. 1. Proposed geometry for the observation of AC conductance oscillations. The electrons are confined to a [001] plane in high mobility GaAs, and conduction channels are created by photolithography along the directions indicated. The electric field (represented by cross-hatching) vanishes in the bent branches; in the horizontal branch, it points in the [001] direction [9,16]. As discussed in the text, electrons that pass through the horizontal branch acquire an additional spin-dependent AC phase δ compared to those that pass through the bent branch. The conductance should oscillate as δ is varied.

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